D-Branes

D 膜

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Abstract

摘要

This is an introduction to the non-perturbative excitations of string theory known as D-branes. Topics covered include their definition and main properties, their role in dualities, and their dyamics. Based on lectures given at LACES 2019 and at the Amsterdam-Brussels-Geneva-Paris Doctoral School.

本文介绍被称为 D 膜的弦论非微扰激发。涵盖主题包括 D 膜的定义、主要性质，其在对偶中的作用，以及 D 膜动力学。内容基于作者 2019 年 LACES 课程与阿姆斯特丹-布鲁塞尔-日内瓦-巴黎博士生学院的授课讲义整理而成。

Keywords

关键词

D-branes - String theory - Dualities - Solitons - Supergravity - Gauge theories

D 膜 - 弦理论 - 对偶性 - 孤子 - 超引力 - 规范理论

Introduction

引言

The discovery of D-branes [1] revolutionized string theory. Although their existence was anticipated in earlier works, see e.g., [2-6], it was the realization that they are exact solitonic excitations of closed-string theory that unleashed a series of developments in a variety of subjects including supersymmetric gauge theories, dualities, string compactifications, quantum black holes, and the gauge/gravity or AdS/CFT correspondence. These developments are covered elsewhere in the present volume, so I will only touch upon them occasionally and briefly. The purpose of this contribution is to provide the background material that is necessary for the study of these more advanced topics.

D 膜的发现 [1] 彻底变革了弦论。尽管早在之前的研究中就有人预言了它的存在，参见例如 [2-6]，但正是“D 膜是闭弦理论的精确孤子激发”这一认知，催生了一系列横跨多个领域的发展，这些领域包括超对称规范理论、对偶性、弦紧致化、量子黑洞，以及规范/引力对偶 (即 AdS/CFT 对应)。本卷的其他部分已经涵盖了这些发展，因此本文仅会偶尔简要提及它们。本文的目的是提供学习这些更高级课题所需的基础背景知识。

The notes are based on the early review Ref. [7] and on lectures given during several editions of the Amsterdam-Brussels-Geneva-Paris Doctoral School and in the 2019 LACES school in Florence. Most of the material is standard and can be found in many textbooks, in particular [8-12]. The presentation is biased by the expertise and taste (or lack thereof!) of the author. To make the lectures self-contained and to define the notation and conventions, I include a very brief introduction to string theory. This and some other parts of the present chapter will overlap with other contributions to this volume.

这些讲义基于早期综述文献 [7]，以及笔者在多届阿姆斯特丹-布鲁塞尔-日内瓦-巴黎博士生学校、2019 年佛罗伦萨 LACES 学校开设的课程。大部分内容都是标准内容，可在诸多教材中找到，尤其是 [8-12]。本文的叙述偏向笔者自身的专业领域与偏好 (或者说笔者的不足！)。为了让讲义内容自成体系，同时明确记号与约定，笔者在这里对弦论做了一个非常简短的介绍。这部分内容以及本章的其他部分会和本卷的其他撰稿内容有所重叠。

The Free Bosonic String

自由玻色弦

Polyakov and Nambu-Goto Actions

波利亚科夫与南布-后藤作用量

The starting point is the Nambu-Goto action which is proportional to the (pseudo)area of the worldsheet $X^{\mu}(\sigma^{\alpha})$ of the string:

> 我们的出发点是南布-后藤作用量，它正比于弦世界面 $X^{\mu}(\sigma^{\alpha})$ 的 (赝) 面积:

$$S_{\mathrm{NG}}=-T_{\mathrm{F}}\int d^{2}\sigma\left[-\det\left(\partial_{\alpha}X^{\mu}\partial_{\beta}X^{\upsilon}\eta_{\mu\upsilon}\right)\right]^{1/2}. \tag{1}$$

Here, $T_{\mathrm{F}}\equiv(2\pi\alpha')^{-1}$ is the tension of the string, $\mu,\upsilon=0,1\cdots d-1$ and $\alpha,\beta=0,1$. The string moves in flat Minkowski spacetime and its worldsheet is parametrized by σ^{α} . The signature of the metric is $(-+\cdots+)$.

> 此处，$T_{\mathrm{F}}\equiv(2\pi\alpha')^{-1}$ 是弦张力，$\mu,\upsilon=0,1\cdots d-1$ 和 $\alpha,\beta=0,1$ 。弦在平坦闵可夫斯基时空中运动，其世界面由 σ^{α} 参数化。度规的符号为 $(-+\cdots+)$ 。

The above action is classically equivalent to that of d free massless scalar fields coupling minimally to an auxiliary two-dimensional metric $g_{\alpha\beta}$[13][1]

> 上述作用量经典上等价于 d 个自由无质量标量场最小耦合辅助二维度规 $g_{\alpha\beta}$[13][1] 的作用量

$$S_{\text{Polyakov}}=-\frac{T_{\mathrm{F}}}{2}\int d^{2}\sigma\sqrt{-g}g^{\alpha\beta}\partial_{\alpha}X^{\mu}\partial_{\beta}X^{\upsilon}\eta_{\mu\upsilon}. \tag{2}$$

Both actions are invariant under reparametrizations of the worldsheet, $\sigma^{\alpha}\rightarrow\tilde{\sigma}^{\alpha}\left(\sigma^{\beta}\right)$. One may choose conformal coordinates so that $g_{\alpha\beta}=e^{\phi}\eta_{\alpha\beta}$. The Liouville field ϕ drops out of the classical action (2), leading to the field equations $\left(\sigma^{\pm}=\sigma^{0}\pm\sigma^{1}\right)$

> 两个作用量都在世界面重参数化 $\sigma^{\alpha}\rightarrow\tilde{\sigma}^{\alpha}\left(\sigma^{\beta}\right)$ 下不变。我们可以选取共形坐标使得 $g_{\alpha\beta}=e^{\phi}\eta_{\alpha\beta}$ 。刘维尔场 ϕ 从经典作用量 (2) 中消去，得到场方程 $\left(\sigma^{\pm}=\sigma^{0}\pm\sigma^{1}\right)$

$$\partial_{+}\partial_{-}X^{\mu}=0,\ \partial_{\pm}X^{\mu}\partial_{\pm}X^{\upsilon}\eta_{\mu\upsilon}=0. \tag{3}$$

The d equations on the left follow by varying X^{μ} , while the two right-hand-side equations follow from variations of the auxiliary metric:

> 左侧的 d 个方程由对 X^{μ} 变分得到，而右侧两个方程由对辅助度规变分得到:

$$\frac{2}{\sqrt{-g}}\frac{\delta S_{\text{Polyakov}}}{\delta g^{\alpha\beta}}\equiv T_{\alpha\beta}\propto\partial_{\alpha}X^{\mu}\partial_{\beta}X_{\mu}-\frac{1}{2}g_{\alpha\beta}\left(\partial_{\gamma}X^{\mu}\partial^{\gamma}X_{\mu}\right)=0. \tag{4}$$

These equations impose that the energy-momentum tensor $T_{\alpha\beta}$ of "matter" fields is zero. Because the trace vanishes identically, there are only two nontrivial equations. Furthermore, the conservation $\partial^{\alpha}T_{\alpha\beta}=0$ means that they need only to be imposed at some initial time. They are therefore phase-space constraints, familiar from the Hamiltonian formulation of gravity. In string theory, they are called the Virasoro conditions.

这些方程要求“物质”场的能量动量张量 $T_{\alpha\beta}$ 为零。由于迹恒为零，因此只有两个非平凡方程。此外，守恒律 $\partial^{\alpha} T_{\alpha\beta} = 0$ 意味着这些方程仅需要在某个初始时刻满足。因此它们是相空间约束，这在引力的哈密顿表述中十分常见，在弦论中它们被称为维拉索罗条件。

Had we started instead with the Nambu-Goto action (1), we could again choose conformal coordinates such that the induced metric $\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \propto \eta_{\alpha\beta}$. The classical equations are then again (3) with the Virasoro constraints now arising as gauge-fixing conditions. The two actions are thus classically equivalent.

如果我们从南布-后藤作用量 (1) 出发，依然可以选取共形坐标使得诱导度规满足 $\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \propto \eta_{\alpha\beta}$ 。经典方程仍然是 (3) 式，此时维拉索罗约束作为规范固定条件出现。因此两个作用量在经典层面是等价的。

Note that one may add to the Polyakov action an Einstein term $-\Phi_0 \int \frac{d^2\sigma}{4\pi} \sqrt{-g} R$, but $\sqrt{g} R = -\nabla^2 \phi$ is a total derivative that does not affect the Virasoro conditions. This term plays nevertheless an important role because

注意，我们可以在波利亞科夫作用量中添加一个爱因斯坦项 $-\Phi_0 \int \frac{d^2\sigma}{4\pi} \sqrt{-g} R$ ，不过 $\sqrt{g} R = -\nabla^2 \phi$ 是全导数，不会影响维拉索罗条件。但这个项仍然有着重要作用，因为

$$\chi_\Sigma \equiv \frac{1}{4\pi} \int_\Sigma \sqrt{g} R = 2 - 2h_\Sigma - b_\Sigma, \tag{5}$$

where h_Σ is the number of handles and b_Σ the number of boundaries of the Riemann surface Σ . Different worldsheet topologies are thus weighted with different powers of the string coupling $g_s \equiv \exp(\Phi_0)$. The topological invariant χ_Σ is called the Euler characteristic. It equals 2 for the sphere, 1 for the disk, and 0 for the torus.

其中 h_Σ 是亏格 (柄的数量)，b_Σ 是黎曼曲面 Σ 的边界数。不同的世界面拓扑会被赋予弦耦合常数 $g_s \equiv \exp(\Phi_0)$ 的不同幂次作为权重。拓扑不变量 χ_Σ 被称为欧拉示性数。它对球面等于 2，对圆盘等于 1，对环面等于 0。

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[1] This action is called the Polyakov action, demonstrating in the words of Polyakov himself [14] the Arnold theorem that ”things are never called after their true inventors.” The trick was indeed first used by J. Douglas in the 1920s to study minimal surfaces. But it is for the quantum string, and as we will see later for the superstring, that it becomes instrumental.

[1] 这个作用量被称为波利亞科夫作用量，正如波利亞科夫本人在文献 [14] 中所说，这印证了阿诺德定理:“事物永远不会以真正发明者的名字命名”。这个技巧确实早在 20 世纪 20 年代就由 J. 道格拉斯研究极小曲面时率先使用了，但它是在量子弦，并且正如我们后续会看到的，在超弦中才变得至关重要。

Classical Motion of Open and Closed Strings

开弦与闭弦的经典运动

The wave equation $\partial_+\partial_- X = 0$ is solved by a sum of left- and right-moving waves, $X = X_R(\sigma^-) + X_L(\sigma^+)$. For closed strings, $\sigma^1 \equiv \sigma^1 + 2\pi$ is periodically identified and the most general solution reads

波动方程 $\partial_+\partial_- X = 0$ 可分解为左行波与右行波之和求解，即 $X = X_R(\sigma^-) + X_L(\sigma^+)$ 。对于闭弦，$\sigma^1 \equiv \sigma^1 + 2\pi$ 具有周期性，最一般的解形式为

$$\text{closed}: \; X^\mu(\sigma) = x^\mu + \alpha' P^\mu \sigma^0 + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n}\left(a_n^\mu e^{-in\sigma^-} + \tilde{a}_n^\mu e^{-in\sigma^+}\right) \tag{6}$$

with n running over nonzero integers. The normalizations have been fixed so that $P^\mu = T_F \int_0^{2\pi} d\sigma^1 \partial_0 X^\mu$ is the center-of-mass momentum of the string, and the canonical Poisson brackets imply $\{a_n^\mu, a_m^\nu\} = \{\tilde{a}_n^\mu, \tilde{a}_m^\nu\} = in\delta_{n+m,0}\delta^{\mu\nu}$. Reality requires $\left(a_n^\mu\right)^* = a_{-n}^\mu$, and likewise for the tilde variables.

其中 n 取遍非零整数。归一化已固定，因此 $P^\mu = T_F \int_0^{2\pi} d\sigma^1 \partial_0 X^\mu$ 是弦的质心动量，由正则泊松括号可得 $\{a_n^\mu, a_m^\nu\} = \{\tilde{a}_n^\mu, \tilde{a}_m^\nu\} = in\delta_{n+m,0}\delta^{\mu\nu}$ 。实性条件要求 $\left(a_n^\mu\right)^* = a_{-n}^\mu$ ，带波浪号的变量同理。

The Virasoro constraints can be solved explicitly in light-cone gauge:

Virasoro 约束可以在光锥规范下显式求解:

$$X^+ = \alpha' P^+ \sigma^0 \;\; \text{and} \;\; \partial_\pm X^- = \frac{2}{\alpha' P^+} \sum_{j=2}^{d-1} \partial_\pm X^j \partial_\pm X^j \tag{7}$$

with $X^\pm = X^0 \pm X^1$. Note that by choosing $X_L^+ = \frac{1}{2}\alpha' P^+ \sigma^+$ and $X_R^+ = \frac{1}{2}\alpha' P^+ \sigma^-$, we removed the residual freedom under coordinate transformations that preserve the conformal gauge, $\sigma^+ \to f(\sigma^+)$ and $\sigma^- \to \tilde{f}(\sigma^-)$. The phase space of a closed string is therefore parametrized by the center-of-mass positions and momenta and by the amplitudes of oscillation in the transverse dimensions. These are only subject to the mass-shell and level-matching conditions which are obtained by integrating Eq. (7) around the string:

其中 $X^\pm = X^0 \pm X^1$ 。注意，通过选择 $X_L^+ = \frac{1}{2}\alpha' P^+ \sigma^+$ 和 $X_R^+ = \frac{1}{2}\alpha' P^+ \sigma^-$ ，我们移除了保持共形规范的坐标变换下的剩余自由度，即 $\sigma^+ \to f(\sigma^+)$ 和 $\sigma^- \to \tilde{f}(\sigma^-)$。因此闭弦的相空间可由质心位置、动量以及横向维度的振荡振幅参数化，这些参数仅满足质壳条件与能级匹配条件，这两个条件可通过对式 (7) 沿弦积分得到:

$$\text{closed}: \; M^2 = -P^\mu P_\mu = \frac{2}{\alpha'} \sum_{j=2}^{d-1} \sum_{n\neq 0} a_{-n}^j a_n^j = \frac{2}{\alpha'} \sum_{j=2}^{d-1} \sum_{n\neq 0} \tilde{a}_{-n}^j \tilde{a}_n^j \, . \tag{8}$$

The reader may wonder why not go to the center-of-mass frame and choose the more intuitive temporal gauge $X^0 = \sigma^0$. The reason is that in this gauge the Virasoro conditions are quadratic in all fields:

读者可能会疑惑为什么不进入质心系并选择更直观的时间规范 $X^0 = \sigma^0$ 。原因是在该规范下，Virasoro 条件对所有场都是二次的:

$$|\partial_+ \mathbf{X}|^2 = |\partial_- \mathbf{X}|^2 = \frac{1}{4} \text{ where } \mathbf{X} = \left(X^1, \cdots X^{d-1}\right), \tag{9}$$

making it hard to eliminate the redundant oscillation amplitudes. Furthermore, as we will see in the next subsection, some states of the quantized string are massless. Gauge (9) is nevertheless useful for proving that cusps, which are strong emitters of gravitational waves [15], would be a generic feature of cosmic strings if these exist [16]. Cusps are points that move at the speed of light, which is equivalent to $\partial_+ \mathbf{X} = \partial_- \mathbf{X}$. But $\partial_\pm \mathbf{X}\left(\sigma^\pm\right)$ trace curves on the round sphere as their arguments vary, so they generically intersect at some (σ^+, σ^-) Q.E.D.

这使得冗余振荡振幅很难被消去。此外，正如我们将在下一小节看到的，量子化弦的部分态是无质量的。不过规范 (9) 仍可用于证明: 如果宇宙弦存在，作为引力波强辐射源的尖点会是宇宙弦的一般特征 [15][16]。尖点是光速运动的点，等价于 $\partial_+ \mathbf{X} = \partial_- \mathbf{X}$ 。但当参数变化时，$\partial_\pm \mathbf{X}\left(\sigma^\pm\right)$ 在单位球面上描出曲线，因此它们通常会在某个 (σ^+, σ^-) 处相交，证毕。

After this amusing digression, we turn now to open strings. For worldsheets $\sum$ with boundary $\partial \sum$ the variation of the Polyakov action in conformal gauge gives

这段有趣的题外话结束后，我们现在转到开弦。对于世界面 $\sum$ 带有边界 $\partial \sum$ 的情况，共形规范下 Polyakov 作用量的变分给出

$$\begin{aligned} \delta S_{\text{Polyakov}} &\propto \int_\sum d^2\sigma \partial^\alpha \left(\delta X_\mu\right) \partial_\alpha X^\mu \\ &= \int_{\partial \sum} d\sigma^\alpha \varepsilon_{\alpha\beta} \partial^\beta X^\mu \delta X_\mu - \int_\sum d^2\sigma \delta X_\mu \partial^\alpha \partial_\alpha X^\mu = 0. \end{aligned} \tag{10}$$

Free endpoints, i.e., those for which δX^μ is arbitrary, must therefore obey Neumann conditions, $\partial_\perp X^\mu = 0$ where $\perp$ stands for normal to the boundary. The most general solution with Neumann conditions at both ends reads [2]

自由端点，即 δX^μ 任意的端点，因此必须满足诺伊曼条件 $\partial_\perp X^\mu = 0$ ，其中 $\perp$ 代表边界法向。两端都满足诺伊曼条件的最一般解为 [2]

$$\text{open } (\mathbf{NN}): \; X^\mu = x^\mu + 2\alpha' P^\mu \sigma^0 + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} a_n^\mu \left(e^{-in\sigma^-} + e^{-in\sigma^+}\right), \tag{11}$$

where $n \in \mathbb{Z}$ and $\sigma^1 \in [0, \pi]$. String excitations are reflected at the endpoints and form standing waves, which is why left- and right-moving amplitudes are identified. Note that when $\partial_1 X^\mu = 0$ the Virasoro conditions reduce to $\partial_0 X^\mu \partial_0 X^\upsilon \eta_{\mu\upsilon} = 0$, which shows that free endpoints travel at the speed of light.

其中 $n \in \mathbb{Z}$ 和 $\sigma^1 \in [0, \pi]$ 。弦激发在端点处反射并形成驻波，这就是左行振幅与右行振幅等价的原因。注意当 $\partial_1 X^\mu = 0$ 时，Virasoro 条件约化为 $\partial_0 X^\mu \partial_0 X^\upsilon \eta_{\mu\upsilon} = 0$ ，这说明自由端点以光速运动。

Another natural boundary condition is the Dirichlet condition which fixes the position of the string endpoints. A relativistic violin string, for example, would have $\mathbf{X}(\sigma^1 = 0) = \mathbf{x}$ and $\mathbf{X}(\sigma^1 = \pi) = \mathbf{x} + \Delta\mathbf{x}$. The mode expansion would in this case read

另一种自然边界条件是固定弦端点位置的狄利克雷边界条件。例如，相对论小提琴弦满足 $\mathbf{X}(\sigma^1 = 0) = \mathbf{x}$ 和 $\mathbf{X}(\sigma^1 = \pi) = \mathbf{x} + \Delta\mathbf{x}$ ，这种情况下模展开式可写为

$$\text{open(DD):}\ \ X^j = x^j + \frac{\sigma^1}{\pi}\Delta x^j + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}a_n^j\left(e^{-in\sigma^-} - e^{-in\sigma^+}\right)\ . \tag{12}$$

Comparing with Eq. (11), we see that the center-of-mass position and momentum is here replaced by $\mathbf{x}$ and $\Delta\mathbf{x}$. Both are fixed, so they are not part of the open-string phase space.

与式 (11) 对比可知，此处质心位置与动量被替换为 $\mathbf{x}$ 和 $\Delta\mathbf{x}$ ，二者都是固定量，因此不属于开弦相空间。

If the X^μ obey Neumann conditions at both ends for $\mu = 0, 1, \cdots, p$ and Dirichlet conditions for $\mu = p+1, \cdots, d-1$, then the string endpoints are forced to move on static parallel p -dimensional hyperplanes, as illustrated in Fig. 1 below.

若 X^μ 在两个端点处对 $\mu = 0, 1, \cdots, p$ 满足诺伊曼边界条件，对 $\mu = p+1, \cdots, d-1$ 满足狄利克雷边界条件，则弦端点被约束在静止平行的 p 维超平面上运动，如下图 1 所示。

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[2] The choice of parametrization is such that the canonical Poisson brackets are the same for both open- and closed-string oscillation amplitudes.

[2] 该参数化选择使得正则泊松括号对开弦和闭弦的振荡振幅保持一致。

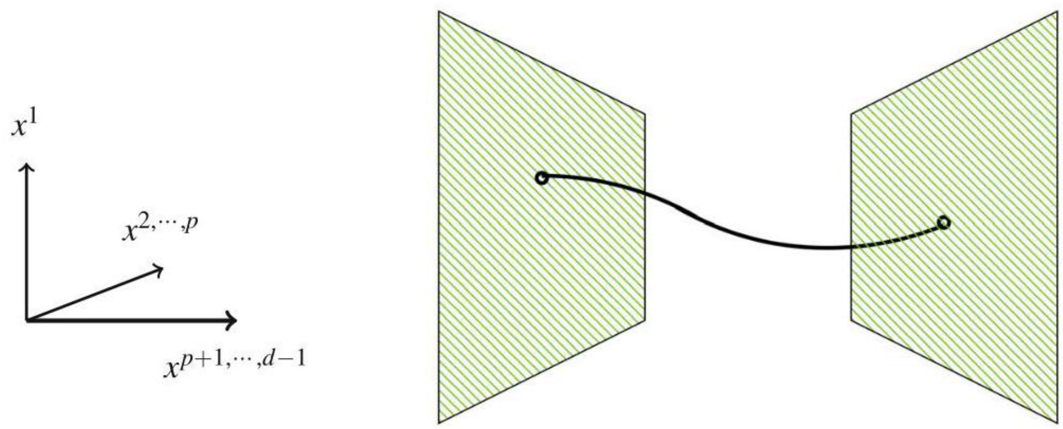


Fig. 1 An open string stretching between two parallel Dp branes

图 1 一根伸展在两个平行 Dp 膜之间的开弦

One can think of these hyperplanes as extended p -dimensional objects. They will turn out to be dynamical, non-perturbative excitations of string theory dubbed Dp branes. Freely moving open strings live on D$(d-1)$ branes, whereas our relativistic violin string is attached to two D0 branes, alias D-particles.

> 我们可以将这些超平面视为延展的 p 维物体，它们实际上是弦论的动力学非微扰激发，被命名为 Dp 膜。自由运动的开弦生活在 D$(d-1)$ 膜上，而我们刚才讨论的相对论小提琴弦则连接在两个 D0 膜 (即 D 粒子) 上。

The mass-shell condition for open strings is obtained as before by integrating Eq. (7). For a string stretching between static parallel Dp branes, it reads

> 开弦的质量壳条件可以通过积分式 (7) 得到，和之前的方法一致。对于伸展在静止平行 Dp 膜之间的弦，质量壳条件为

$$\text{open }(\mathbf{D}p\mathbf{D}p):\ M^2 = -P^\mu P_\mu = \frac{1}{2\alpha'}\sum_{j=2}^{d-1}\sum_{n\neq 0} a^j_{-n}a^j_n + |T_F\Delta\mathbf{x}|^2\,, \tag{13}$$

where μ runs over the $p+1$ Neumann directions only. The last term on the right can be recognized as the mass squared of an open string stretching linearly between the two D-branes. It enters in the mass formula like momentum in some hidden dimension. As we will later see, this is not a coincidence but a consequence of a deep symmetry of string theory called T-duality. Comparing with Eq. (8) suggests that a closed string is the same as two open strings tied together. There is actually more to this remark than meets the eye.

> 其中 μ 仅遍历 $p+1$ 诺伊曼方向。右侧最后一项可识别为线性伸展在两个 D 膜之间的开弦的质量平方，它在质量公式中的作用类似于某个隐藏维度中的动量。我们在后文会看到，这并非巧合，而是弦论中名为 T 对偶的深刻对称性的结果。和式 (8) 对比可知，闭弦等价于连接在一起的两根开弦，这个结论实际上暗藏更多深意。

More generally, the two D-branes at the string endpoints can be different. They may have, in particular, a different orientation, uniform relative motion, or different dimension. In all these situations, the boundary conditions remain linear and the problem can be readily solved [17, 18]. As an example, consider two straight static D1 branes, alias D-strings, which make an angle ϑ in the (x^1, x^2) plane as illustrated in Fig. 2. Let the first D-string be oriented along the x^1 direction so that X^1 obeys a Neumann condition and X^2 a Dirichlet condition at $\sigma^1 = 0$. At the other endpoint, $\sigma^1 = \pi$, the coordinate $(X^1\cos\vartheta + X^2\sin\vartheta)$ obeys a Neumann condition, while the coordinate $(X^1\sin\vartheta - X^2\cos\vartheta)$ obeys a Dirichlet condition. To avoid confusion, the stretched open string is often referred to as an F-string ('F' for fundamental). The general solution with these boundary condition is given by Eq. (12) for all $j = 3, \cdots, d-1$ since these are Dirichlet at both endpoints and by

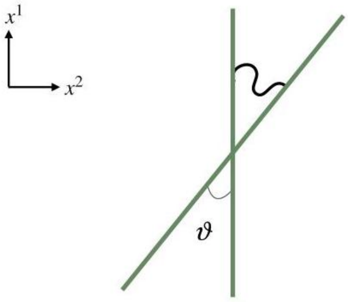
更一般地说，弦端点处的两个 D 膜可以不同，它们尤其可能具有不同取向、匀速相对运动或者不同维度。在所有这些情形下，边界条件仍然是线性的，问题可以直接求解 [17, 18]。举个例子，考虑两张直的静态 D1 膜，也就是 D 弦，它们在 (x^1, x^2) 平面上形成夹角 ϑ ，如图 2 所示。设第一张 D 弦沿 x^1 方向取向，因此在 $\sigma^1 = 0$ 处，X^1 满足诺依曼边界条件，X^2 满足狄利克雷边界条件。在另一端点 $\sigma^1 = \pi$ 处，坐标 $(X^1 \cos\vartheta + X^2 \sin\vartheta)$ 满足诺依曼边界条件，而坐标 $(X^1 \sin\vartheta - X^2 \cos\vartheta)$ 满足狄利克雷边界条件。为避免混淆，这根伸展的开弦通常被称为 F 弦 (F 代表基本弦)。满足这些边界条件的通解可由式 (12) 给出，对所有满足两端狄利克雷条件的 $j = 3, \cdots, d-1$ 适用，通解也可由

$$X^1 + iX^2 = \sqrt{\frac{\alpha'}{2}}\left[\sum_r \frac{1}{r}\left(a_r e^{-ir\sigma^-} + a_r^* e^{ir\sigma^+}\right) + \sum_s \frac{1}{s}\left(b_s^* e^{is\sigma^-} + b_s e^{-is\sigma^+}\right)\right],$$

(14)

Fig. 2 An open F-string between two straight D-strings making an angle ϑ

图 2 两张成夹角 ϑ 的直 D 弦之间的开 F 弦



where the oscillation frequencies are now $r \in \mathbb{Z} + \frac{\vartheta}{\pi}$ and $s \in \mathbb{Z} - \frac{\vartheta}{\pi}$. One can check indeed that at $\sigma^1 = 0$ the complex coordinate $X^1 + iX^2$ is real so that X^2 obeys a Dirichlet condition, while at $\sigma^1 = \pi$, it is the combination $e^{-i\vartheta}(X^1 + iX^2)$ that becomes real as required by the Dirichlet condition $X^1 \sin\vartheta - X^2 \cos\vartheta = 0$. Note that for $\vartheta = \pi/2$, i.e., for mixed Dirichlet-Neumann boundary conditions, all oscillation frequencies are half-integer.

其中振荡频率现在为 $r \in \mathbb{Z} + \frac{\vartheta}{\pi}$ 和 $s \in \mathbb{Z} - \frac{\vartheta}{\pi}$ 。我们确实可以验证，在 $\sigma^1 = 0$ 处复坐标 $X^1 + iX^2$ 为实数，因此 X^2 满足狄利克雷条件；而在 $\sigma^1 = \pi$ 处，组合 $e^{-i\vartheta}(X^1 + iX^2)$ 为实数，符合狄利克雷条件 $X^1 \sin\vartheta - X^2 \cos\vartheta = 0$ 的要求。注意对于 $\vartheta = \pi/2$ ，也就是混合狄利克雷-诺依曼边界条件，所有振荡频率都是半整数。

The expansion (14) has no zero mode, so the center of mass of the F-string is localized at the intersection of the D-strings, at $X^1 = X^2 = 0$. Escaping to infinity requires infinite energy and is forbidden.

(14) 式的展开没有零模，因此 F 弦的质心局域在 D 弦的交点，位置为 $X^1 = X^2 = 0$ 。逃逸到无穷远需要无穷能量，因此是禁戒的。

Quantization

量子化

In the classical theory, the amplitudes a_n^i are continuous complex variables, and the spectrum, Eq. (8) or Eq. (13), is continuous and positive. In the quantum theory, the amplitudes become operators that obey the commutation relations:

经典理论中，振幅 a_n^i 是连续复变量，谱 (式 (8) 或式 (13)) 是连续且恒正的。量子理论中，振幅成为满足下列对易关系的算符:

$$\left[a_n^i, \left(a_m^j\right)^\dagger\right] = \left[\tilde{a}_n^i, \left(\tilde{a}_m^j\right)^\dagger\right] = n\delta_{n-m,0}\delta^{ij}. \tag{15}$$

The ground state is annihilated by all $n > 0$ operators, while the conjugate operators $a_{-n}^j = \left(a_n^j\right)^\dagger$ and $\tilde{a}_{-n}^j = \left(\tilde{a}_n^j\right)^\dagger$ create excited string states. Furthermore, normal ordering the infinite sums in Eq. (8) or Eq. (13) shifts the spectrum by an infinite constant which must be renormalized.

所有 $n > 0$ 算符湮灭基态，而共轭算符 $a_{-n}^j = \left(a_n^j\right)^\dagger$ 和 $\tilde{a}_{-n}^j = \left(\tilde{a}_n^j\right)^\dagger$ 产生激发弦态。此外，对式 (8) 或式 (13) 中的无穷和做正规序会使谱偏移一个无穷常数，该常数必须被重整化。

For concreteness, we consider the open string attached to two parallel D p branes. Upon quantization, Eq. (13) becomes

具体而言，我们考虑连接两块平行 D p 膜的开弦。量子化后，式 (13) 变为

$$\text{open } (\mathbf{D}p\mathbf{D}p): \ \alpha' M^2 = \hat{N} + \frac{|\Delta \mathbf{x}|^2}{4\pi^2\alpha'} - \frac{d-2}{24}, \tag{16}$$

where $\hat{N}$ is the normal-ordered ”level operator”

其中 $\hat{N}$ 是正规序后的“能级算符”

$$\hat{N} = \sum_{j=2}^{d-1}\sum_{n>0} a_{-n}^j a_n^j \Rightarrow \left[\hat{N}, a_n^\dagger\right] = n a_n^\dagger \text{ and } \left[\hat{N}, a_n\right] = -n a_n, \tag{17}$$

and the last term in (16) is the zero-point contribution. This latter comes from a collection of harmonic oscillators, one for each transverse coordinate and for each frequency $\hbar\omega = n$. Using the zeta-function regularization, we find

式 (16) 的最后一项是零点贡献。该项来自一组简谐振子，每个横坐标、每个频率 $\hbar\omega = n$ 对应一个振子。利用 ζ 函数正则化，我们得到

$$(d-2)\sum_{n=1}^{\infty}\frac{n}{2} = \frac{(d-2)}{2}\zeta(-1) = -\frac{(d-2)}{24} \tag{18}$$

which is the result announced in Eq. (16).

这就是式 (16) 给出的结果。

It is important that the zeta function respects locality on the string worldsheet. To see why, let $\sigma^1 \in [0, \pi L]$, where L is some irrelevant worldsheet-distance scale which will drop out in the end from the expression for M^2 . The zero-point contribution to $\alpha' M^2/L$, regularized by a short-distance cutoff ε , reads

ζ 函数满足弦世界面的局域性，这一点十分重要。为说明原因，设 $\sigma^1 \in [0, \pi L]$ ，其中 L 是无关的世界面距离标度，最终会从 M^2 的表达式中消去。由短距离截断 ε 正则化后的零点对 $\alpha' M^2/L$ 的贡献为

$$\sum_{n=1}^{\infty} \frac{n}{2L} e^{-n\varepsilon/L} = \frac{L}{2\varepsilon^2} - \frac{1}{24L} + O\left(\frac{\varepsilon^2}{L^2}\right).$$

A local subtraction must be proportional to L , so it removes precisely the leading $\varepsilon \to 0$ term Q.E.D.

局域减除必须正比于 L ，因此恰好消去领头项 $\varepsilon \to 0$ ，证毕。

It is straightforward to extend the analysis to open strings stretching between nonidentical D-branes. Each (NN) or (DD) coordinate contributes as above, while each complex coordinate like (14) has modes with fractional frequencies $n \pm \frac{\vartheta}{\pi}$ and gives a zero-point contribution:

不难将该分析推广到延伸在不同 D 膜之间的开弦。每个 (NN) 或 (DD) 坐标的贡献和上述相同，而如式 (14) 的每个复坐标带有分数频率 $n \pm \frac{\vartheta}{\pi}$ 的模式，给出的零点贡献为:

$$\frac{1}{2}\sum_{n=1}^{\infty}\left(n - \frac{|\vartheta|}{\pi}\right) + \frac{1}{2}\sum_{n=0}^{\infty}\left(n + \frac{|\vartheta|}{\pi}\right) = -\frac{1}{12} + \frac{1}{2}\left(\frac{\vartheta}{\pi}\right)^2. \tag{19}$$

Here, $\vartheta \in (-\pi, \pi)$, and to obtain this result we regularized and subtracted the sum as previously explained. Note that for $\vartheta = \frac{\pi}{2}$ the result is $\frac{1}{24}$, so the zero-point contribution of each (ND) or (DN) coordinate is $\frac{1}{48}$.

此处 $\vartheta \in (-\pi, \pi)$ ，我们得到该结果的方法是按前文说明对求和做正则化和减除。注意当 $\vartheta = \frac{\pi}{2}$ 时结果为 $\frac{1}{24}$ ，因此每个 (ND) 或 (DN) 坐标的零点贡献为 $\frac{1}{48}$ 。

Closed strings have periodic coordinates which are integer modded. Level operators can be defined separately in the left-moving and right-moving sectors and they are matched by Eq. (8). After quantization, these constraints read

闭弦具有模为整数的周期坐标。可分别在左行和右行扇区定义能级算符，二者由式 (8) 匹配。量子化后，这些约束条件写为

$$\text{closed}: \ \frac{1}{4}\alpha' M^2 = \widehat{N}_L - \frac{d-2}{24} = \widehat{N}_R - \frac{d-2}{24}\,. \tag{20}$$

Winding, which is analogous to $\Delta \mathbf{x}$, and fractionally modded coordinates arise when space is compactified on tori or on orbifolds. In these cases, Eq. (20) must be appropriately modified.

当空间在环面或轨形上紧致化时，会出现类似 $\Delta\mathbf{x}$ 的缠绕和模为分数的坐标，此时式 (20) 需要做适当修正。

Mass Spectra and Critical Dimension

质量谱与临界维数

The mass eigenstates of the string are constructed by acting with creation operators, $\left(a_n^j\right)^\dagger$ with $n > 0$, on the ground state $|0\rangle$. From the commutation relations (17), we see that the level of a state is the sum of the frequencies of all the creation operators applied to the ground state. We have also seen that $\alpha' M^2 = \hat{N} + \alpha' M_0^2$ for open strings and $\frac{1}{4}\alpha' M^2 = \hat{N}_L + \alpha' M_0^2 = \hat{N}_R + \alpha' M_0^2$ for closed strings, with M_0^2 (or $4M_0^2$) the mass squared of the ground state.

弦的质量本征态由产生算子作用在基态 $|0\rangle$ 上构造得到，$\left(a_n^j\right)^\dagger$ 满足 $n > 0$ 。由对易关系 (17) 可知，一个态的能级等于作用在基态上所有产生算子的频率之和。我们还已知开弦满足 $\alpha' M^2 = \hat{N} + \alpha' M_0^2$ ，闭弦满足 $\frac{1}{4}\alpha' M^2 = \hat{N}_L + \alpha' M_0^2 = \hat{N}_R + \alpha' M_0^2$ ，其中 M_0^2 (或 $4M_0^2$)) 是基态的质量平方。

Let us first discuss an open string with freely moving endpoints, i.e., ”attached” to space-filling D $(d-1)$ branes. Mass eigenstates should transform in representations of the unbroken Lorentz symmetry $SO(1, d-1)$. The lowest lying state is a scalar with mass $\alpha' M_0^2 = -(d-2)/24$. This is negative for $d > 2$, so the state is a tachyon. A theory with spontaneously broken gauge symmetry has a tachyon if expanded around the symmetric vacuum. Tachyons are bad but not a priori fatal - they may just signal a wrong choice of the vacuum.

我们首先讨论端点自由运动的开弦，即“连接”在全空间填充 D $(d-1)$ 膜上的开弦。质量本征态应按照未破缺洛伦兹对称性 $SO(1, d-1)$ 的表示变换。最低能态是质量为 $\alpha' M_0^2 = -(d-2)/24$ 的标量。当 $d > 2$ 时该质量为负，因此这个态是快子。如果在对称真空附近展开，自发破缺规范对称性的理论会存在快子。快子本身有问题但并非完全不可救药——它可能只是说明我们选错了真空。

The first excited states $a_{-1}^j|0\rangle$ of the open string, in light-cone quantization, form a vector of $SO(d-2) \subset SO(1, d-1)$. This is the little group of a massless spinning particle - a massive one has one more polarization state. Changing the vacuum cannot save the day since the number of degrees of freedom is preserved. To avoid a contradiction, these states must thus be massless:

在光锥量子化下，开弦的第一激发态 $a_{-1}^j|0\rangle$ 构成 $SO(d-2) \subset SO(1, d-1)$ 的矢量。这对应无质量自旋粒子的小群——有质量粒子会多一个极化态。改变真空也无法解决这个问题，因为自由度数目是守恒的。为了避免矛盾，这些态必须是无质量的:

$$0 = 1 - \frac{(d-2)}{24} \Rightarrow d = 26. \tag{21}$$

This is the critical dimension of the bosonic string. When $d \neq 26$, the algebra of the Lorentz-group generators $J^{\mu\upsilon} = \int d\sigma^1 \left(X^\mu \partial_0 X^\upsilon - X^\upsilon \partial_0 X^\mu\right)$ has a quantum anomaly and the symmetry is broken. [3]

这就是玻色弦的临界维数。当 $d \neq 26$ 时，洛伦兹群生成元的代数 $J^{\mu\upsilon} = \int d\sigma^1 \left(X^\mu \partial_0 X^\upsilon - X^\upsilon \partial_0 X^\mu\right)$ 会出现量子反常，对称性被破坏。[3]

There are no further problems at the higher levels. At level two, for example, one finds a vector $a^j_{-2}|0\rangle$ and a 2-index symmetric tensor $a^j_{-1}a^k_{-1}|0\rangle$ of $SO(24)$. They combine nicely into a symmetric traceless 2-index tensor of $SO(25)$ - the little group for a massive particle. These facts are summarized in Table 1.

更高能级不存在其他问题。例如二级能级可以得到 $SO(24)$ 的一个矢量 $a^j_{-2}|0\rangle$ 和一个二阶对称张量 $a^j_{-1}a^k_{-1}|0\rangle$ 。它们可以恰好组合成有质量粒子小群 $SO(25)$ 的对称无迹二阶张量。这些结果总结在表 1 中。

The interested reader can consult Ref. [19] for a discussion of how massive string states organize into $SO(25)$ representations. Here, we will only count their number, $\mathcal{N}\left(\hat{N}\right)$, at each level $\hat{N}$. For one transverse coordinate, this would have been the number of partitions of $\hat{N}$ into positive integers, with generating function $\sum \mathcal{N}\left(\hat{N}\right) q^{\hat{N}} = \prod_{n=1}^{\infty} (1-q^n)^{-1}$. For several coordinates, the generating functions multiply. Using also the relation $\alpha' M^2 = \hat{N} - 1$, we finally get

感兴趣的读者可查阅文献 [19]，其中讨论了有质量弦态如何组织为 $SO(25)$ 表示。本文仅统计每个能级 $\hat{N}$ 上的态数 $\mathcal{N}\left(\hat{N}\right)$ 。对单个横向坐标，态数就是将 $\hat{N}$ 拆分为正整数的分拆数，其生成函数为 $\sum \mathcal{N}\left(\hat{N}\right) q^{\hat{N}} = \prod_{n=1}^{\infty} (1-q^n)^{-1}$ 。多个坐标的生成函数为各坐标生成函数的乘积。结合关系式 $\alpha' M^2 = \hat{N} - 1$ ，我们最终得到

$$Z(q) = \sum_{\hat{N}=0}^{\infty} \mathcal{N}\left(\hat{N}\right) q^{\alpha' M^2} = q^{-1} \prod_{n=1}^{\infty} (1-q^n)^{-24} = q^{-1} + 24 + 324q + \cdots \tag{22}$$

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[3] The anomaly arises in the commutator of two J^{k-} . In the alternative covariant quantization the missing states are provided by the Liouville mode which does not decouple when $d \neq 26$ [13]. Theories with a dynamical Liouville mode are called non-critical string theories.

[3] 该反常出现在两个 J^{k-} 的对易子中。在替代的协变量子化中，缺失的态由刘维尔模提供，当 $d \neq 26$ 时刘维尔模不退耦 [13]。包含动力学刘维尔模的理论被称为非临界弦理论。

Table 1 The first three levels of the bosonic open string with freely moving endpoints in $d = 26$ and the corresponding representation of the Poincaré group

表 1 玻色开弦端点自由运动时，$d = 26$ 中前三个能级及其对应庞加莱群的表示

$\alpha' M^2$	States	Representation
-1	0□	Scalar
0	$a^J_{-1}\|0\rangle$	Transverse vector
1	$a^j_{-1}a^k_{-1}\|0\rangle, a^j_{-2}\|0\rangle$	Symmetric traceless tensor

Note that $Z(q)$ should not be confused with the canonical partition function of the string in which states are weighted with the Boltzmann factor $\exp(-\beta M)$. The number of states of a highly excited string can be extracted from the saddle point approximation with the result

请注意，不要将 $Z(q)$ 与弦的正则配分函数混淆，后者中态会被玻尔兹曼因子 $\exp(-\beta M)$ 加权。高度激发弦的态数可通过鞍点近似提取，结果为

$$\mathcal{N}\left(\hat{N}\right) = \oint \frac{dq}{2\pi i} q^{-\hat{N}} Z(q) \sim e^{4\pi\sqrt{\hat{N}}}. \tag{23}$$

To perform the calculation, one writes $Z(q)$ as a power of Dedekind’s eta function:

为完成计算，可将 $Z(q)$ 写为戴德金 η 函数的幂次:

$$Z(q) = \eta(q)^{-24} \quad \text{where} \quad \eta(q) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n). \tag{24}$$

This function, which we will encounter later in the cylinder amplitude, transforms as a modular form of weight 1/2 under fractional linear transformations of τ , where $q = \exp(2\pi i\tau)$. Under inversion of τ in particular

该函数我们之后会在柱振幅中遇到，它在 τ 的分式线性变换下按权为 1/2 的模形式变换，其中 $q = \exp(2\pi i\tau)$ 。特别地，在 τ 反演下

$$\eta\left(-\frac{1}{\tau}\right) = \sqrt{-i\tau}\eta(\tau). \tag{25}$$

To obtain the estimate (23), one uses that fact that for large $\hat{N}$ the contour integral is dominated by a saddle point at small $\operatorname{Im}\tau$ where

为得到估计式 (23)，我们利用以下事实: 当 $\hat{N}$ 很大时，围道积分由小 $\operatorname{Im}\tau$ 处的鞍点主导，该处

$$\eta(\tau) = \sqrt{-i\tau}\eta\left(-\frac{1}{\tau}\right) \sim e^{-i\pi/12\tau}.$$

The linear growth of the entropy with mass, $\mathcal{N}\left(\hat{N}\right) \sim \exp\left(4\pi\sqrt{\alpha'}M\right)$, means that the canonical partition function of a free string cannot be defined beyond the limiting Hagedorn temperature $\beta_H = 4\pi\sqrt{\alpha'}$.

熵随质量线性增长，即 $\mathcal{N}\left(\hat{N}\right) \sim \exp\left(4\pi\sqrt{\alpha'}M\right)$ ，意味着自由弦的正则配分函数在极限哈格多恩温度 $\beta_H = 4\pi\sqrt{\alpha'}$ 之外无法定义。

It is straightforward to extend the above analysis to closed strings. From Eq. (20), we see that in the critical dimension $d = 26$ the ground state is a tachyon with $\alpha' M^2 = -4$. The first excited states $\alpha^j_{-1}\tilde{\alpha}^k_{-1}|0\rangle$ are massless. They transform as a general 2-index tensor of $SO(24)$, whose traceless symmetric, antisymmetric, and trace parts are identified in the second-quantized string theory with fluctuations of the space-time metric $G_{\mu\nu}$, an antisymmetric field $B_{\mu\nu}$, and a scalar field Φ . We will discuss these in more detail after introducing the superstring. More generally, the states of a closed string transform as tensor products of two copies of the open string at level $\hat{N} = \hat{N}_L = \hat{N}_R$. Since $\mathcal{N}_{\text{closed}}\left(\hat{N}\right) = \mathcal{N}_{\text{open}}\left(\hat{N}\right)^2$ and $M_{\text{closed}} = 2M_{\text{open}}$ at any given level, the Hagedorn temperature stays the same. This is to be expected because the high-energy fluctuations of the string should depend only mildly on boundary conditions.

可以很直接地将上述分析推广到闭弦。从式 (20) 中我们可以看出，在临界维度 $d = 26$ 下，基态是一个满足 $\alpha' M^2 = -4$ 的快子。第一激发态 $\alpha^j_{-1}\tilde{\alpha}^k_{-1}|0\rangle$ 是无质量的，它们变换为 $SO(24)$ 的广义二阶张量，在二次量子化弦论中，该张量的无迹对称部分、反对称部分和迹部分分别对应时空度规 $G_{\mu\nu}$ 的涨落、反对称场 $B_{\mu\nu}$ 和标量场 Φ 。我们会在引入超弦后对这些内容展开更详细的讨论。更一般地说，闭弦的态可变换为能级 $\hat{N} = \hat{N}_L = \hat{N}_R$ 处两份开弦态的张量积。由于在任意给定能级都满足 $\mathcal{N}_{\text{closed}}\left(\hat{N}\right) = \mathcal{N}_{\text{open}}\left(\hat{N}\right)^2$ 和 $M_{\text{closed}} = 2M_{\text{open}}$ ，哈格多恩温度保持不变。这一结论符合预期，因为弦的高能量涨落对边界条件的依赖本就很微弱。

Superstrings

超弦

Supersymmetry cures the problem of the tachyon and introduces string states that transform in spinor representations of the Lorentz group, i.e., they are spacetime fermions. We will employ the covariant Neveu-Schwarz-Ramond (NSR) formalism following at first closely the classic textbook [20] which predates D-branes. Readers interested in the alternative Green-Schwarz formulation of the superstring can also consult this textbook.

超对称性解决了快子问题，并引入了按洛伦兹群的旋量表示变换的弦态，也就是说这些弦态是时空费米子。我们将采用协变的纳沃-施瓦茨-拉蒙德 (NSR) 形式体系，一开始会紧密遵循早于 D 膜出现的经典教科书 [20]。对超弦的另一种格林-施瓦茨表述感兴趣的读者也可以查阅这本教科书。

Worldsheet Supergravity

世界面超引力

The Polyakov action Eq. (2) describes a conformal field theory (CFT) coupled to gravity in two (worldsheet) dimensions. Gravity is non-dynamical and serves to impose the vanishing of the energy-momentum tensor, $T_{\alpha\beta} = 0$. Since $T_{\alpha\beta}$ is traceless for a CFT[4] . In critical string theories, this anomaly cancels between ”matter” and ghost fields. we are left with two equations, the Virasoro conditions.

Polyakov 作用量式 (2) 描述了一个耦合二维 (世界面) 引力的共形场论 (CFT)。引力是非动力学的，其作用是要求能量动量张量为零，$T_{\alpha\beta} = 0$ 。因为 $T_{\alpha\beta}$ 对 CFT[4] 而言是无迹的。在临界弦论中，该反常会在“物质”场与鬼场之间抵消，最终我们得到两个方程，即 Virasoro 条件。

Superstring theory can be likewise described as two-dimensional supergravity coupled to a superconformal field theory (SCFT). The simplest SCFT has a free massless boson X and a free massless Majorana fermion ψ with action

超弦理论同样可以描述为耦合超共形场论 (SCFT) 的二维超引力。最简单的超共形场论包含一个自由无质量玻色子 X 和一个自由无质量马约拉纳费米子 ψ，其作用量为

$$S = \int d^2\sigma \left(-\frac{1}{2}\partial_\alpha X \partial^\alpha X + \frac{i}{2}\overline{\psi}\rho^\alpha \partial_\alpha \psi\right). \tag{26}$$

We set for now $2\pi\alpha' = 1$. The Dirac algebra $\{\rho^\alpha, \rho^\beta\} = -2\eta^{\alpha\beta}$ can be represented by purely imaginary matrices in two dimensions:

我们目前设 $2\pi\alpha' = 1$ 。狄拉克代数 $\{\rho^\alpha, \rho^\beta\} = -2\eta^{\alpha\beta}$ 在二维空间中可以用纯虚矩阵表示:

$$\rho^0 = \begin{pmatrix} 0 - i \\ i\ 0 \end{pmatrix},\ \rho^1 = \begin{pmatrix} 0\ i \\ i\ 0 \end{pmatrix}. \tag{27}$$

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[4] On curved worldsheets, the Weyl anomaly implies $T^\alpha_\alpha = \frac{c}{12}R$ where R is the Ricci scalar and c the central charge of the ”matter” CFT ($c = 1$ for a free scalar field and 1/2 for a free Majorana fermion)

[4] 在弯曲世界面上，魏尔反常意味着 $T^\alpha_\alpha = \frac{c}{12}R$ ，其中 R 是里奇标量，c 是“物质”共形场论的中心荷 (自由标量场为 ($c = 1$ ，自由马约拉纳费米子为 1/2)

The Lorentz-boost generator $J^{01} = \frac{i}{2}\rho^0\rho^1$ is thus also imaginary and the spinor $\psi = (\psi_R, \psi_L)$ is real. From Dirac’s equation

洛伦兹 boost 生成元 $J^{01} = \frac{i}{2}\rho^0\rho^1$ 因此也是虚数，旋量 $\psi = (\psi_R, \psi_L)$ 是实旋量。根据狄拉克方程

$$0 = \rho^\alpha \partial_\alpha \psi = \begin{pmatrix} 0 & -2i\partial_- \\ 2i\partial_+ & 0 \end{pmatrix} \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} \tag{28}$$

we see that ψ_R is a function of σ^- and ψ_L is a function of σ^+ . The action (26) is invariant under the global supersymmetry transformations:

我们可以看到 ψ_R 是 σ^- 的函数，ψ_L 是 σ^+ 的函数。作用量 (26) 在整体超对称变换下不变:

$$\delta X = \varepsilon^T \rho^0 \psi,\ \delta\psi = -i\rho^\alpha \partial_\alpha X \varepsilon \tag{29}$$

with ε a Majorana spinor. [5] Noether’s theorem gives the conserved supercurrent

其中 ε 是马约拉纳旋量。[5] 诺特定理给出守恒超流

$$J_\alpha = -\frac{1}{2}\rho^\beta \rho_\alpha \psi \partial_\beta X \tag{30}$$

which carries (in addition to the vector) a spinor index that was here suppressed. Thus, J has four components, but two of them vanish automatically by virtue of the identity $\rho^\alpha\rho^\beta\rho_\alpha = 0$ which implies $\rho^\alpha J_\alpha = 0$. This is the supersymmetric partner of the zero-trace condition $T_\alpha{}^\alpha = 0$ for a SCFT. The remaining components can be recast into the combinations:

它 (除矢量外) 还带有一个旋量指标，此处省略。因此，J 共有四个分量，但由于恒等式 $\rho^\alpha\rho^\beta\rho_\alpha = 0$ ，其中两个分量自动为零，该恒等式还推出 $\rho^\alpha J_\alpha = 0$ 。这是超共形场论零迹条件 $T_\alpha{}^\alpha = 0$ 的超对称伙伴。剩余分量可以改写为以下组合:

$$J_+ = \frac{1}{2}(J_0 + J_1) = \begin{pmatrix} 0 \\ \psi_L \partial_+ X \end{pmatrix} \text{ and } J_- = \frac{1}{2}(J_0 - J_1) = \begin{pmatrix} \psi_R \partial_- X \\ 0 \end{pmatrix}. \tag{31}$$

If we endow X and ψ with a spacetime-vector index μ , as in the bosonic string, the nonvanishing supercurrents read $\psi_L^\mu \partial_+ X_\mu$ and $\psi_R^\mu \partial_- X_\mu$. The idea is to couple the SCFT to non-dynamical supergravity in order to impose the constraints $J_\pm = 0$ from an action principle [21].

如果我们和玻色弦一样，给 X 和 ψ 赋予一个时空矢量指标 μ ，非零超流就是 $\psi_L^\mu \partial_+ X_\mu$ 和 $\psi_R^\mu \partial_- X_\mu$ 。我们的思路是将超共形场论与非动力学超引力耦合，从而从作用量原理出发得到约束条件 $J_\pm = 0$ [21]。

Let us see how this works. Coupling a spinor to gravity requires an orthonormal frame (or zweibein) e_a^α where a is a flat-space index. We have $e_a^\alpha e_b^\beta g_{\alpha\beta} = \eta_{ab}$ and $e_\alpha^a e_\beta^b \eta_{ab} = g_{\alpha\beta}$ for the inverse zweibein. Spinors transform under Lorentz rotations of the local frame, but they are scalars under diffeomorphisms. Their coupling to gravity is through the spin connection $\omega_{\alpha b}^a$, which can be expressed in terms of the zweibein if one insists that the frame be covariantly constant:

我们来看具体的构造方式。将旋量耦合到引力需要正交标架 (或双标架 zweibein) e_a^α ，其中 a 是平直空间指标。我们有 $e_a^\alpha e_b^\beta g_{\alpha\beta} = \eta_{ab}$ ，逆双标架满足 $e_\alpha^a e_\beta^b \eta_{ab} = g_{\alpha\beta}$ 。旋量在局部标架的洛伦兹转动下变换，但在微分同胚下是标量。它们通过自旋联络 $\omega_{\alpha b}^a$ 耦合到引力，如果要求标架是协变常数的，自旋联络可以用双标架表示:

$$0 = D_\alpha e_\beta^a \Rightarrow \omega_{\alpha b}^a = e_b^\beta \left(\partial_\alpha e_\beta^a - \Gamma_{\alpha\beta}^\gamma e_\gamma^a\right). \tag{32}$$

Here, $\Gamma_{\alpha\beta}^\gamma$ is the affine connection, and the spin connection ω_α^{ab} is antisymmetric in the flat indices (ab). The coupling of the free massless multiplet to gravity reads

在此，$\Gamma_{\alpha\beta}^\gamma$ 是仿射联络，自旋联络 ω_α^{ab} 在平坦指标 (ab) 下是反对称的。无质量自由多重态与引力的耦合可写为

$$S_2 = -\frac{1}{2}\int d^2\sigma\sqrt{-g}\left(g^{\alpha\beta}\partial_\alpha X \partial_\beta X - i\overline{\psi}\rho^\alpha D_\alpha \psi\right), \tag{33}$$

[5] This is called $N = (1, 1)$ supersymmetry because there are two transformations acting separately on the left-moving and right-moving sectors.

[5] 这被称为 $N = (1,1)$ 超对称，因为存在两个变换分别作用于左行和右行扇区。

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where $D_\alpha\psi \equiv \left(\partial_\alpha + \frac{1}{4}\omega_\alpha^{ab}\rho_{ab}\right)\psi, \rho_{ab} \equiv \frac{1}{2}[\rho_a, \rho_b]$, and $\rho^a = e_\alpha^a\rho^\alpha$ are the flat-space Dirac matrices. Under local supersymmetry transformations

其中 $D_\alpha\psi \equiv \left(\partial_\alpha + \frac{1}{4}\omega_\alpha^{ab}\rho_{ab}\right)\psi, \rho_{ab} \equiv \frac{1}{2}[\rho_a, \rho_b]$ 和 $\rho^a = e_\alpha^a\rho^\alpha$ 是平坦空间狄拉克矩阵。在局域超对称变换下

$$\delta S_2 = -2\int d^2\sigma\sqrt{-g}\left(D_\alpha\bar{\varepsilon}\right)J^\alpha. \tag{34}$$

This can be cancelled by the variation of a gravitino term:

这可以通过引力微子项的变分抵消:

$$S_3 = 2\int d^2\sigma\sqrt{-g}\overline{\chi}_\alpha J^\alpha \tag{35}$$

with $\delta\chi_\alpha = D_\alpha\varepsilon$. But this is not the end of the story because J^α transforms under local supersymmetry transformations. The extra variation is however cancelled by the additional term:

其中 $\delta\chi_\alpha = D_\alpha\varepsilon$ 。但故事并未到此结束，因为 J^α 会在局域超对称变换下发生变换。不过额外的变分可通过附加项抵消:

$$S_4 = -\frac{1}{4}\int d^2\sigma\sqrt{-g}\left(\overline{\psi}\psi\right)\left(\overline{\chi}_\alpha\rho^\beta\rho^\alpha\chi_\beta\right), \tag{36}$$

and by modifying the supersymmetry transformations as follows:

并对超对称变换做如下修改:

$$\delta X = \bar{\varepsilon}\psi,\ \delta\psi = -i\rho^\alpha\varepsilon\left(\partial_\alpha X - \overline{\psi}\chi_\alpha\right)$$

$$\delta e_\alpha^a = -2i\bar{\varepsilon}\rho^a\chi_\alpha,\ \delta\chi_\alpha = D_\alpha\varepsilon. \tag{37}$$

The action $S_2 + S_3 + S_4$ is invariant under diffeomorphisms, plus local Lorentz and supersymmetry transformations. It has in addition three "accidental" local symmetries. The Weyl symmetry already encountered in the Polyakov string:

作用量 $S_2 + S_3 + S_4$ 在微分同胚、局域洛伦兹变换和超对称变换下保持不变。此外它还有三个“偶然”局域对称性。就是我们在波利雅科夫弦中已经见过的外尔对称性:

$$X \to X,\ \psi \to \Omega^{-1/2}\psi,\ e_\alpha^a \to \Omega e_\alpha^a,\ \chi_\alpha \to \Omega^{1/2}\chi_\alpha, \tag{38}$$

and its super-extension which only transforms the gravitino field

以及它的超推广，该推广仅变换引力微子场

$$\delta \chi_\alpha = i\rho_\alpha \eta,\ \delta(\text{ rest }) = 0. \tag{39}$$

Using local supersymmetry, one can set $\chi_\alpha = i\rho_\alpha \chi$. This is the extension of the conformal gauge for the bosonic string, with χ the superpartner of the Liouville field ϕ . Thanks to superconformal invariance, ϕ and χ drop out from the classical supergravity action since the corresponding components of the energy-momentum tensor and supercurrent, T_{+-}, J_{+R}, J_{-L} , are identically zero.

利用局域超对称，我们可以令 $\chi_\alpha = i\rho_\alpha \chi$。这是玻色弦共形规范的推广，其中 χ 是刘维尔场 ϕ 的超对称伴侣。借助超共形不变性，ϕ 和 χ 会从经典超引力作用量中消去，因为能量动量张量和超流的对应分量 T_{+-}, J_{+R}, J_{-L} 恒为零。

After the dust has settled, the entire supergraviton multiplet can be gauge fixed away leaving us with free fields (X^μ, ψ^μ) . The supergravity equations impose the vanishing of the remaining four components of $T_{\alpha\beta}$ and J_α on the initial data. More explicitly, these "super-Virasoro" conditions read

尘埃落定后，整个超引力子多重态都可以通过规范固定消去，仅留下自由场 (X^μ, ψ^μ)。超引力方程要求 $T_{\alpha\beta}$ 和 J_α 的剩余四个分量在初始数据上为零。更明确地说，这些“超-维拉索罗”条件可写为

$$\eta_{\mu v}\left(\partial_+ X^\mu \partial_+ X^v + \frac{i}{2}\psi_L^\mu \partial_+ \psi_L^v\right) = \eta_{\mu v}\left(\partial_- X^\mu \partial_- X^v + \frac{i}{2}\psi_R^\mu \partial_- \psi_R^v\right) = 0 \tag{40}$$

$$\text{and } \eta_{\mu\nu}\psi_L^\mu \partial_+ X^\nu = \eta_{\mu\nu}\psi_R^\mu \partial_- X_\nu = 0\,. \tag{41}$$

They can be solved explicitly by going to the light-cone gauge, as we did for the bosonic string in section "The Free Bosonic String." Indeed, using the residual freedom that is not fixed by the superconformal gauge, one can set [6]

就像我们在章节“自由玻色弦”中对玻色弦做的那样，可以通过进入光锥规范显式求解它们。事实上，利用超共形规范没有固定的剩余自由度，我们可以令 [6]

$$X^+ = \alpha' P^+ \sigma^0 \ \text{ and }\ \psi_L^+ = \psi_R^+ = 0, \tag{42}$$

whre $X^\pm = X^0 \pm X^1$ and $\psi^\pm = \psi^0 \pm \psi^1$. With this choice, Equations (40) and (41) become linear in X^- and ψ^- . They can be used to express these coordinates in terms of the independent data $\left\{X^j, \psi^j\right\}$ for $j = 2, \cdots, d-1$.

其中 $X^\pm = X^0 \pm X^1$ 和 $\psi^\pm = \psi^0 \pm \psi^1$。在此选择下，方程 (40) 和 (41) 对 X^- 和 ψ^- 是线性的。我们可以利用它们将这些坐标用独立数据 $\left\{X^j, \psi^j\right\}$ (对应 $j = 2, \cdots, d-1$) 表示出来。

Neveu-Schwarz and Ramond Sectors

内沃-施瓦茨扇区与拉蒙德扇区

The coordinates X^μ of the superstring have the same mode expansions as those of the bosonic string, and they obey the same canonical commutation relations (15). So let us focus on the fermionic coordinates. For closed superstrings, the left and right modes are independent:

超弦的坐标 X^μ 与玻色弦有着相同的模展开，且满足相同的正则对易关系 (15)。因此我们将重点放在费米坐标上。对于闭超弦，左行模与右行模相互独立:

$$\text{closed}: \quad (\psi_R^\mu, \psi_L^\mu) = \left(\sum_r \psi_r^\mu e^{-ir\sigma^-}, \sum_{\tilde{r}} \tilde{\psi}_{\tilde{r}}^\mu e^{-i\tilde{r}\sigma^+} \right). \tag{43}$$

Reality of the Majorana spinors implies $\left(\psi_r^\mu\right)^\dagger = \psi_{-r}^\mu$ and $\left(\tilde{\psi}_{\tilde{r}}^\mu\right)^\dagger = \tilde{\psi}_{-\tilde{r}}^\mu$ and the canonical anti-commutation relations read

马约拉纳旋量的实性给出约束 $\left(\psi_r^\mu\right)^\dagger = \psi_{-r}^\mu$ 和 $\left(\tilde{\psi}_{\tilde{r}}^\mu\right)^\dagger = \tilde{\psi}_{-\tilde{r}}^\mu$，正则反对易关系为

$$\left\{\psi_r^\mu, \psi_s^\upsilon\right\} = \left\{\tilde{\psi}_r^\mu, \tilde{\psi}_s^\upsilon\right\} = \delta_{r+s,0}\eta^{\mu\upsilon}. \tag{44}$$

Here comes now an important new feature. Since all observables contain an even number of fermions, the ψ^μ can have either periodic or antiperiodic conditions. Thus, the mode frequencies r and $\tilde{r}$ in the expansions (43) can a priori be either all integer or all half-integer. We must however make sure that the two supercurrents $J_{-R} = \psi_R^\mu \partial_- X_\mu$ and $J_{+L} = \psi_L^\mu \partial_+ X_\mu$, which generate residual gauge symmetries, are well defined modulo a sign. This means that

这里出现了一个重要的新特性: 由于所有可观测量都包含偶数个费米子，因此 ψ^μ 可以满足周期性或反周期性边界条件。因此，展开式 (43) 中的模频率 r 和 $\tilde{r}$ 先验地要么全为整数，要么全为半整数。但我们需要保证生成剩余规范对称性的两个超流 $J_{-R} = \psi_R^\mu \partial_- X_\mu$ 和 $J_{+L} = \psi_L^\mu \partial_+ X_\mu$ 在符号差下是良定义的。这意味着

$$J_{-R}\left(\sigma^1 + 2\pi\right) = \eta_R J_{-R}\left(\sigma^1\right) \quad \text{and} \quad J_{+L}\left(\sigma^1 + 2\pi\right) = \eta_L J_{+L}\left(\sigma^1\right) \tag{45}$$

where η_L, η_R are signs. Put differently, all the ψ_R^μ must be simultaneously periodic or antiperiodic and likewise for the ψ_L^μ. The choices $\eta = +$ and $\eta = -$ are called the Ramond (**R**) and the Neveu-Schwarz (NS) boundary conditions. Since for the closed superstring η_L and η_R are independent, there exist four possibilities: NS-NS, NS-R, R-NS, and R-R. We will see that all four are needed.

其中 η_L, η_R 是符号。换句话说，所有 ψ_R^μ 必须同时为周期性或同时为反周期性，ψ_L^μ 同理。这两种选择 $\eta = +$ 和 $\eta = -$ 分别被称为拉蒙德 (**R**) 边界条件和内沃-施瓦茨 (NS) 边界条件。由于闭超弦的 η_L 和 η_R 相互独立，因此共有四种可能:NS-NS、NS-R、R-NS 和 R-R，我们后续会看到这四种情况都是必要的。

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[6] The residual supersymmetry transformations correspond to arbitrary $\varepsilon_R\left(\sigma^-\right)$ and $\varepsilon_L\left(\sigma^+\right)$. From $\delta\psi^+ = -i\rho^+\varepsilon\partial_+ X^+ - i\rho^-\varepsilon\partial_- X^+ = \alpha' p^+\left(\varepsilon_R, -\varepsilon_L\right)$, we see that these suffice to set the left-and right-moving components of the on-shell fermion ψ^+ to zero.

[6] 剩余超对称变换对应任意的 $\varepsilon_R(\sigma^-)$ 和 $\varepsilon_L(\sigma^+)$。从 $\delta\psi^+ = -i\rho^+\varepsilon\partial_+X^+ - i\rho^-\varepsilon\partial_-X^+ = \alpha' p^+(\varepsilon_R, -\varepsilon_L)$ 可以看出，我们足以通过变换将壳费米子 ψ^+ 的左行和右行分量都置零。

What about the open superstrings? Varying the action of a Majorana fermion on an open worldsheet produces a boundary term:

开超弦的情况如何呢？对开世界面上的马约拉纳费米子作用量变分会得到一个边界项:

$$\delta S = \frac{i}{2}\int_{\partial\sum} d\sigma^\alpha \varepsilon_{\alpha\beta}\psi^T\rho^0\rho^\beta\delta\psi. \tag{46}$$

Taking the boundary along the σ^0 direction, this variation vanishes provided that

当边界沿 σ^0 方向取时，只要满足下式，该变分就等于零

$$\psi^T\rho^0\rho^1\delta\psi = \psi_R\delta\psi_R - \psi_L\delta\psi_L|_{\partial\sum} = 0 \;\Leftrightarrow\; \psi_R = \pm\psi_L|_{\partial\sum}. \tag{47}$$

The left- and right-moving components of the Majorana field are therefore identified on the boundary up to a sign. As in the case of the closed string, we must however make sure that the boundary conditions preserve superconformal invariance. [7] The existence of a conserved supercurrent requires that $J_{+L} = \eta J_{-R}$ on $\partial\sum$ or explicitly

因此马约拉纳场的左行分量和右行分量在边界上相差一个符号等同。和闭弦的情况一样，我们必须保证边界条件保留超共形不变性。[7] 守恒超流的存在要求在 $\partial\sum$ 上满足 $J_{+L} = \eta J_{-R}$，即显式地

$$\psi_L^\mu\partial_+X_\mu = \eta\psi_R^\mu\partial_-X_\mu|_{\partial\sum}. \tag{48}$$

It follows that $\psi_L^\mu = \eta\psi_R^\mu$ if X^μ obeys a Neumann condition and $\psi_L^\mu = -\eta\psi_R^\mu$ when X^μ obeys a Dirichlet condition.

由此可得: 当 X^μ 满足诺依曼条件时，$\psi_L^\mu = \eta\psi_R^\mu$ 成立；当 X^μ 满足狄利克雷条件时，$\psi_L^\mu = -\eta\psi_R^\mu$ 成立。

An open string has two boudaries, at $\sigma^1 = 0, \pi$. Since one sign can be absorbed by redefining the supercurrent, only the relative sign $\eta_0\eta_\pi \equiv \eta$ is relevant. There are thus only two sectors of the open superstring, the Neveu-Schwarz sector $(\eta = -)$ and the Ramond sector $(\eta = +)$. Choosing $\eta_0 = +$ leads to the following mode expansion of fermionic coordinates with (NN) or (DD) boundary conditions:

开弦有两个边界，位于 $\sigma^1 = 0, \pi$ 。由于一个符号可以被超流重新定义吸收，因此只有相对符号 $\eta_0\eta_\pi \equiv \eta$ 是相关的。因此开超弦仅有两个 sector: 诺依福-施瓦茨 sector $(\eta = -)$ 和拉蒙德 sector $(\eta = +)$。选择 $\eta_0 = +$ 后，可得到满足 (NN) 或 (DD) 边界条件的费米坐标如下模式展开:

$$\text{open}: \quad \left(\psi_R^\mu, \psi_L^\mu\right) = \begin{cases} \sum_r \psi_r^\mu \left(e^{-ir\sigma^-}, e^{-ir\sigma^+}\right) \\ \sum_r \psi_r^\mu \left(e^{-ir\sigma^-}, -e^{-ir\sigma^+}\right) \end{cases} \tag{49}$$

with $r \in \mathbb{Z}$ in the Ramond sector and $r \in \mathbb{Z} + \frac{1}{2}$ in the Neveu-Schwarz sector.

其中拉蒙德 sector 为 $r \in \mathbb{Z}$，诺依福-施瓦茨 sector 为 $r \in \mathbb{Z} + \frac{1}{2}$。

These expansions are all one needs for parallel identical Dp branes. The analysis can be extended readily to the case of open superstrings stretching between different D-branes. In a nutshell, the fermionic coordinate goes along with its bosonic partner for the ride. If this latter obeys the generic boundary condition Eq. (14), the boundary condition of $\psi^1 + i\psi^2$ is the same at $\sigma^1 = 0$ and the same up to the sign η at $\sigma^1 = \pi$. Thus, in the Ramond sector, fermionic and bosonic modes have the same mode frequencies, while in the Neveu-Schwarz sector, the fermionic ones are shifted compared to the bosonic ones by a factor of $\frac{1}{2}$.

对于平行的等同 Dp 膜，这些展开已经足够使用。该分析可以很容易地推广到伸展在不同 D 膜之间的开超弦情形。简言之，费米坐标会随其玻色伙伴一同变换。若玻色坐标满足通式边界条件 (14), 则 $\psi^1 + i\psi^2$ 的边界条件在 $\sigma^1 = 0$ 处保持不变，在 $\sigma^1 = \pi$ 处仅相差符号 η。因此，拉蒙德 sector 中费米模和玻色模具有相同的模频率，而诺依福-施瓦茨 sector 中，费米模的频率比玻色模偏移了 $\frac{1}{2}$。

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[7] This is necessary because superconformal invariance is a residual gauge symmetry. There is no such requirement for global symmetries which may be broken by the boundary conditions. An example is Poincaré symmetry which is (partially) broken by localized D-branes.

[7] 这一要求是必要的，因为超共形不变性是剩余规范对称性。而整体对称性没有这个要求，它可以被边界条件破缺。比如庞加莱对称性就会被局域化的 D 膜 (部分) 破缺。

Without further ado, we turn now to the spectrum of superstrings, beginning with the closed ones.

言归正传，我们现在来讨论超弦的谱，先从闭弦开始。

GSO and Type-II Superstrings

GSO 投影与 II 型超弦

The mass-shell and level-matching conditions for a closed superstring are obtained by integrating the Virasoro constraints (40):

闭超弦的质量壳条件和能级匹配条件可通过积分 Virasoro 约束 (40) 得到:

$$\text{closed} \quad : \quad \left.\begin{array}{r}(\text{NS})\,\hat{N}_R-\frac{d-2}{16}\\ (\mathbf{R})\,\hat{N}_R\end{array}\right\}=\frac{\alpha'}{4}M^2=\left\{\begin{array}{ll}\hat{N}_L-\frac{d-2}{16} & (\text{N})\\ \hat{N}_L & (\text{L}\end{array}\right. \qquad (\text{NS})$$

(50) (R)

where the level operators that measure the total oscillator frequency read

其中测量总振荡频率的能级算符为

$$\hat{N}_R=\sum_{j=2}^{d-1}\left(\sum_{n>0}\alpha_{-n}^j\alpha_n^j+\sum_{r>0}r\psi_{-r}^j\psi_r^j\right), \tag{51}$$

and likewise for $\hat{N}_L$ with $\left\{\alpha_n^j,\psi_r^j\right\}$ replaced by the left-moving amplitudes $\left\{\tilde{\alpha}_n^j,\tilde{\psi}_r^j\right\}$. The sum over r runs over positive integers in the Ramond sector and over positive half-integers in the Neveu-Schwarz sector. Zero-point oscillations cancel between bosons and fermions for Ramond, while for Neveu-Schwarz, one finds

将 $\left\{\alpha_n^j,\psi_r^j\right\}$ 替换为左行振幅 $\left\{\tilde{\alpha}_n^j,\tilde{\psi}_r^j\right\}$ 即可得到 $\hat{N}_L$ 的对应形式。对 r 的求和在 Ramond 区取正整数，在 Neveu-Schwarz 区取正半整数。Ramond 区中玻色子和费米子的零点振荡相互抵消，而 Neveu-Schwarz 区可得

$$(d-2)\left(\sum\frac{n}{2}-\sum\frac{r}{2}\right)=(d-2)\left(-\frac{1}{24}-\frac{1}{48}\right)=-\frac{d-2}{16}. \tag{52}$$

From Eq. (50), we can now obtain the spectrum of the superstring in each of the four sectors of the worldsheet fermions.

根据式 (50)，我们现在可以得到世界面费米子四个分区中每个分区的超弦谱。

The NS-NS ground state $|0\rangle_{\text{NS}}\otimes|0\rangle_{\text{NS}}$ is a tachyon for $d>2$, and the first excited states $\psi_{-1/2}^j\,|0\rangle_{\text{NS}}\otimes\tilde{\psi}_{-1/2}^k|0\rangle_{\text{NS}}$ transform as a general 2-index tensor of $SO(d-2)$. For consistency, this latter must be the little group of massless particles:

NS-NS 区的基态 $|0\rangle_{\text{NS}}\otimes|0\rangle_{\text{NS}}$ 是 $d>2$ 的快子，第一激发态 $\psi_{-1/2}^j\,|0\rangle_{\text{NS}}\otimes\tilde{\psi}_{-1/2}^k|0\rangle_{\text{NS}}$ 按照 $SO(d-2)$ 的一般二阶张量变换。为了自洽，该二阶张量必须是无质量粒子的小群:

$$0=\frac{1}{2}-\frac{(d-2)}{16}\Rightarrow d=10. \tag{53}$$

Critical superstrings live therefore in ten spacetime dimensions. In the second-quantized theory, each string state corresponds to a spacetime field. The covariant fields that create the above states are the tachyon T, the string-frame metric $G_{\mu\nu}$, the antisymmetric Kalb-Ramond field $B_{\mu\nu}$, and the dilaton Φ. All other NS-NS states are massive and decouple from the effective field theory at energies much below the string scale $\alpha'^{-1/2}$.

因此临界超弦存在于十维时空中。在二次量子化理论中，每个弦态对应一个时空场。生成上述态的协变场分别是快子 T、弦框架度规 $G_{\mu\nu}$、反对称 Kalb-Ramond 场 $B_{\mu\nu}$ 和伸缩子 Φ。所有其他 NS-NS 态都是有质量的，在远低于弦能标 $\alpha'^{-1/2}$ 的能量下会退耦出有效场论。

The lowest states in the R-NS sector are $\left|0\right\rangle_{\rm R} \otimes \widetilde{\psi}^j_{-1/2}\left|0\right\rangle_{\rm NS}$ and they are massless. They obey level matching in the critical dimension $d = 10$ as seen from Eq. (50). In contrast however with the NS ground state which is a scalar, the **R** ground state is not unique but it transforms nontrivially under $SO(8)$. The reason is that it must represent the algebra of fermionic zero modes which commute with $\widehat{N}_L$ and hence with M^2 . The canonical anticommutation relations $\left\{\psi^j_0, \psi^k_0\right\} = \delta^{jk}$ are the same as the algebra of Dirac matrices Γ^j , so the **R** ground states form an $SO(8)$ spinor. We conclude that the massless **R** -NS carry one vector index and one spinor index, so they are spacetime fermions. The corresponding fields are a gravitino Ψ^μ that obeys the trace condition $\Gamma_\mu \Psi^\mu = 0$ and a dilatino Ξ (spinor indices are here suppressed). All other R-NS states are massive, and since they carry one spinor and any number of vector indices, they are all spacetime fermions.

> R-NS 区的最低能态是 $\left|0\right\rangle_{\rm R} \otimes \widetilde{\psi}^j_{-1/2}\left|0\right\rangle_{\rm NS}$，且均为无质量。如式 (50) 所示，它们满足临界维度 $d = 10$ 下的能级匹配。但和作为标量的 NS 基态不同，**R** 基态不唯一，它在 $SO(8)$ 下有非平凡变换。原因是它必须表示与 $\widehat{N}_L$ 对易、因此也与 M^2 对易的费米零模代数。正则对易关系 $\left\{\psi^j_0, \psi^k_0\right\} = \delta^{jk}$ 和狄拉克矩阵代数 Γ^j 形式一致，因此 **R** 基态构成一个 $SO(8)$ 旋量。我们可得结论: 无质量 **R** -NS 态携带一个矢量指标和一个旋量指标，因此它们是时空费米子。对应的场是满足迹条件 $\Gamma_\mu \Psi^\mu = 0$ 的引力微子 Ψ^μ 和伸缩微子 Ξ (此处省略旋量指标)。所有其他 R-NS 态都是有质量的，且由于它们携带一个旋量指标和任意多个矢量指标，因此全都是时空费米子。

The NS-R sector is identical and contributes a second gravitino field $\widetilde{\Psi}^\mu$ and a second dilatino $\widetilde{\Xi}$, plus massive states. Finally, the $\mathbf{R}-\mathbf{R}$ ground states transform as a bispinor of $SO(8)$, i.e., as a matrix with two spinor indices, so they are spacetime bosons. We denote the corresponding bispinor field by **C** . It can be decomposed in terms of n -form gauge fields as we will discuss in a moment.

> NS-R 扇区是相同的，贡献了第二个引力微子场 $\widetilde{\Psi}^\mu$ 和第二个 dilatino(dilatino: 伸缩微子) $\widetilde{\Xi}$，外加有质量态。最终，$\mathbf{R}-\mathbf{R}$ 基态按 $SO(8)$ 的双旋量变换，即作为带有两个旋量指标的矩阵，因此它们是时空玻色子。我们将对应的双旋量场记为 **C** 。它可以按 n 形式规范场分解，我们很快就会讨论这一点。

The theory constructed so far still includes a tachyon. But contrary to the bosonic string, this instability can here be cured by imposing the Gliozzi-Scherk-Olive (GSO) projections [22]. These only keep states of even worldsheet-fermion parity, separately in the left- and in the right-moving sectors. The fermion-parity operators are denoted $(-)^F$ and $(-)^{\widetilde{F}}$, and they obey

> 到目前为止构造的理论仍然包含快子。但与玻色弦不同，这里的不稳定性可以通过施加 Gliozzi-Scherk-Olive(GSO) 投影来解决 [22]。GSO 投影只保留世界面费米子宇称为偶的态，在左行和右行扇区分别进行。费米宇称算子记为 $(-)^F$ 和 $(-)^{\widetilde{F}}$ ，它们满足

$$\left\{(-)^F, \psi^j_r\right\} = \left\{(-)^{\widetilde{F}}, \widetilde{\psi}^j_r\right\} = 0. \tag{54}$$

When strings join or split parity is multiplicative, so only the even projections are consistent in the interacting theory. What is less obvious at first is that the parity of the NS ground state is odd, so that GSO projects indeed out the tachyon. A proper explanation involves the construction of vertex operators for the emission of string states (see, e.g., [8, 12]). The tachyon vertex operator is odd under both $(-)^F$ and $(-)^{\widetilde{F}}$ and is hence projected out as advertized. [8]

当弦合并或分裂时，宇称是可乘的，因此在相互作用理论中只有偶宇称投影是自洽的。最初不太明显的一点是:NS 基态的宇称为奇，因此 GSO 确实会将快子投影出去。合适的解释需要构造弦态发射的顶点算子 (例如参见 [8, 12])。快子顶点算子在 $(-)^F$ 和 $(-)^{\tilde{F}}$ 下都是奇的，因此如预期被投影出去。[8]

On the Ramond ground states, on the other hand, the parity operator acts like the product $\Gamma = \prod_{j=2}^{9} \Gamma^j$. This follows from the anticommutation relations (54) for $r = 0$. The GSO-projected Ramond ground state is therefore a Weyl spinor of $SO(8)$, and it is also Majorana because the fermionic zero modes are real. There exist two inequivalent Weyl-Majorana representations of $SO(8)$ denoted $\mathbf{8}_s$ and $\mathbf{8}_c$.

另一方面，在 Ramond 基态上，宇称算子的作用等价于乘积 $\Gamma = \prod_{j=2}^{9} \Gamma^j$ 。这可以从 $r = 0$ 的对易关系 (54) 推出。因此经 GSO 投影的 Ramond 基态是 $SO(8)$ 的外尔旋量，同时它也是马约拉纳旋量，因为费米零模是实的。$SO(8)$ 存在两个不等价的外尔-马约拉纳表示，记为 $\mathbf{8}_s$ 和 $\mathbf{8}_c$ 。

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[8] More generally, ad hoc truncations of the free-string spectrum violate the symmetry under global reparametrizations of the worldsheet, also called modular invariance, which guarantees ultraviolet finiteness. This excludes, for example, theories with only NS-NS sectors or with no fermion-parity projections at all. One choice that respects modular invariance is to keep only the NS-NS and R-R sectors and to perform an overall projection $(-)^F(-)^{\tilde{F}} = +$. The ensuing theories called type-0A or type-0B have no spacetime fermions, but they are still tachyonic.

[8] 更一般地说，对自由弦谱的随意截断会破坏世界面整体重参数化下的对称性，该对称性也称为模不变性，它保证了紫外有限性。例如，这就排除了仅包含 NS-NS 扇区或完全不做费米宇称投影的理论。有一种选择满足模不变性: 仅保留 NS-NS 和 R-R 扇区，并做整体投影 $(-)^F(-)^{\tilde{F}} = +$ 。由此得到的理论称为 0A 型或 0B 型，它们没有时空费米子，但仍然存在快子。

Which one we declare to have even parity is a matter of convention, it depends on whether we identify $(-)^F$ and $(-)^{\tilde{F}}$ with Γ or with $-\Gamma$. What is physically relevant is the relative chirality of the left- and right-moving Ramond ground states. The theory where the two have opposite chirality is called type IIA, the one with the same chirality is called type IIB.

我们指定哪一个为偶宇称是约定问题，这取决于我们将 $(-)^F$ 和 $(-)^{\tilde{F}}$ 对应为 Γ 还是 $-\Gamma$ 。物理上相关的是左行和右行 Ramond 基态的相对手征性。两个基态手征性相反的理论称为 IIA 型，手征性相同的理论称为 IIB 型。

What do the GSO projections imply for the R-R gauge fields? Acting on the bispinor $\mathbf{C}$, they read

GSO 投影对 R-R 规范场意味着什么？作用在双旋量 $\mathbf{C}$ 上，投影可写为

$$\mathbf{C} = \Gamma\mathbf{C} = \begin{cases} -\mathbf{C}\Gamma & \text{for IIA,} \\ +\mathbf{C}\Gamma & \text{for IIB.} \end{cases} \tag{55}$$

Now, a complete basis of matrices in spinor space is given by all antisymmetric products $\Gamma^{i_1 i_2 \cdots i_n} \equiv \Gamma^{[i_1}\Gamma^{i_2}\cdots\Gamma^{i_n]}$. Thus, the bispinor can be decomposed into a sum of antisymmetric n -form fields:

现在，旋量空间中矩阵的一组完备基由所有反对称乘积 $\Gamma^{i_1 i_2 \cdots i_n} \equiv \Gamma^{[i_1}\Gamma^{i_2}\cdots\Gamma^{i_n]}$ 给出。因此，双旋量可以分解为反对称 n -形式场的和:

$$C_{i_1 i_2 .. i_n} \equiv \operatorname{tr}\left(\mathbf{C}\Gamma_{i_1 i_2 .. i_n}\right). \tag{56}$$

The projections (55) then imply that there are only odd- n forms in the IIA theory and only even- n forms in the IIB theory. In addition, the identities

投影 (55) 由此表明，IIA 理论中仅存在奇 n 形式，IIB 理论中仅存在偶 n 形式。此外，恒等式

$$\Gamma\Gamma_{i_1 i_2 \ldots i_n} = \frac{(-)^{n(n+1)/2}}{(8-n)!}\varepsilon_{i_1 i_2 \ldots i_8}\Gamma^{i_{n+1}\ldots i_8} \tag{57}$$

relate n -forms and $(8-n)$ forms. Thus, the type-IIA theory has independent 1-form and 3-form **R** - **R** fields C_μ and $C_{\mu\upsilon\rho}$, while the type-IIB theory has a 0-form (or scalar) C , a 2-form $C_{\mu\upsilon}$, and a self-dual 4-form $C^{\text{s.d.}}_{\mu\upsilon\rho\tau}$. All this is summarized in Table 2 below.

将 n -形式与 $(8-n)$ 形式联系起来。因此，IIA 型理论具有独立的 1-形式和 3-形式 **R** - **R** 场 C_μ 和 $C_{\mu\upsilon\rho}$ ，而 IIB 型理论具有 0-形式 (即标量) C 、2-形式 $C_{\mu\upsilon}$ 和自对偶 4-形式 $C^{\text{s.d.}}_{\mu\upsilon\rho\tau}$ 。所有内容总结在下表 2 中。

The massless states of the type-II superstrings are in one-to-one correspondence with the fields of the two maximal supergravities in ten dimensions. The non-chiral type-IIA theory can be obtained from the eleven-dimensional supergravity [24] by dimensional reduction. The three NS-NS fields of string theory combine into the eleven-dimensional metric G_{MN} and the two **R**−**R** fields into an antisymmetric 3- form A_{MNR} . The chiral type-IIB supergravity [25,26] does not descend from higher dimensions, and its self-dual 4-form [9] hinders a covariant Lagrangian description. Both theories have 128 bosonic and 128 fermionic on-shell states. This is the content of the maximal supergraviton multiplet whose decomposition in terms of $SO(8)$ representations is $(\mathbf{8}_\upsilon \oplus \mathbf{8}_s)\otimes(\mathbf{8}_\upsilon \oplus \mathbf{8}_c)$ for type-IIA and $(\mathbf{8}_\upsilon \oplus \mathbf{8}_s)\otimes(\mathbf{8}_\upsilon \oplus \mathbf{8}_s)$ for type-IIB, where $\mathbf{8}_\upsilon$ stands for the vector representation.

II 型超弦的无质量态与十维两种最大超引力的场一一对应。非手征的 IIA 型理论可以通过维度约化从十一维超引力 [24] 得到。弦论的三个 NS-NS 场组合为十一维度规 G_{MN} ，两个 **R** − **R** 场组合为反对称 3-形式 A_{MNR} 。手征 IIB 型超引力 [25,26] 并非来自更高维，其自对偶 4-形式 [9] 阻碍了协变拉格朗日描述。两种理论都具有 128 个玻色子和 128 个费米子 on-shell 态。这就是最大超引力子多重态的内容，它按 $SO(8)$ 表示分解为:IIA 型是 $(\mathbf{8}_\upsilon \oplus \mathbf{8}_s)\otimes(\mathbf{8}_\upsilon \oplus \mathbf{8}_c)$ ，IIB 型是 $(\mathbf{8}_\upsilon \oplus \mathbf{8}_s)\otimes(\mathbf{8}_\upsilon \oplus \mathbf{8}_s)$ ，其中 $\mathbf{8}_\upsilon$ 代表矢量表示。

Table 2 The massless fields of the type-II supergravities and the corresponding string states

表 2 II 型超引力的无质量场及对应的弦态

String states	IIA theory	IIB theory
$\psi^j_{-1/2}\,\|0\rangle_{\mathrm{NS}} \otimes \widetilde{\psi}^k_{-1/2}\|\,0\rangle_{\mathrm{NS}}$	$G_{\mu\nu}, B_{\mu\nu}, \Phi$	$G_{\mu\nu}, B_{\mu\nu}, \Phi$
$\|0\rangle_{\mathrm{R}} \otimes \|\,0\rangle_{\mathrm{R}}$	$C_\mu, C_{\mu\upsilon\rho}$	$C, C_{\mu\upsilon}, C^{\mathrm{s.d.}}_{\mu\upsilon\rho\tau}$
$\|0\rangle_{\mathrm{R}} \otimes \widetilde{\psi}^J_{-1/2}\|\,0\rangle_{\mathrm{NS}}$	opposite chirality	same chirality
$\psi^j_{-1/2}\,\|0\rangle_{\mathrm{NS}} \otimes \|\,0\rangle_{\mathrm{R}}$	$\Psi^\mu, \Xi \& \widetilde{\Psi}^\mu, \widetilde{\Xi}$	$\Psi^\mu, \Xi \& \widetilde{\Psi}^\mu, \widetilde{\Xi}$

D-Branes and Spacetime Supersymmetry

D 膜与时空超对称

Superconformal transformations of the string worldsheet are gauge symmetries whose generators annihilate physical states. We argued in section "Neveu-Schwarz and Ramond Sectors" that boundaries must preserve a diagonal subgroup of these symmetries in order to eliminate negative-norm states from the open-string Hilbert space.

弦世界面的超共形变换是规范对称性，其生成元湮灭物理态。我们在“内沃-施瓦茨 sector 与拉蒙德 sector”一节中已经指出，开弦希尔伯特空间要消除负范数态，边界就必须保留这些对称性的一个对角子群。

There is no such restriction for global worldsheet symmetries which may be completely broken by the boundary conditions. Consider, for example, the symmetry under translations of a bosonic coordinate X . The corresponding Noether charge, $\oint d\sigma^\alpha \varepsilon_{\alpha\beta}\mathfrak{J}^\beta$ with $\mathfrak{J}_\alpha = \partial_\alpha X$, is conserved on a closed worldsheet. There is actually a second current, $\mathfrak{W}^\alpha = \varepsilon_{\alpha\beta}\mathfrak{J}^\beta$, whose continuity equation $\partial_\alpha \mathfrak{W}^\alpha = 0$ is an identity, so that the corresponding charge $\oint d\sigma^\alpha \mathfrak{J}_\alpha$ is also conserved. These two charges are the center-of-mass momentum and winding of a closed string, both nontrivial when x is a compact dimension.

对于整体世界面对称性没有这类限制，它们可以被边界条件完全破缺。举例来说，考虑玻色坐标 X 的平移对称性。对应的带 $\mathfrak{J}_\alpha = \partial_\alpha X$ 的诺特荷 $\oint d\sigma^\alpha \varepsilon_{\alpha\beta}\mathfrak{J}^\beta$ 在封闭世界面上守恒。实际上还存在第二个流 $\mathfrak{W}^\alpha = \varepsilon_{\alpha\beta}\mathfrak{J}^\beta$ ，其连续性方程 $\partial_\alpha \mathfrak{W}^\alpha = 0$ 是一个恒等式，因此对应的荷 $\oint d\sigma^\alpha \mathfrak{J}_\alpha$ 也守恒。这两个荷分别是闭弦的质心动量和缠绕，当 x 是紧致维度时二者都是非平庸的。

Now, consider an open worldsheet. For a contractible contour, $\oint d\sigma^\alpha \varepsilon_{\alpha\beta}\mathfrak{J}^\beta = 0$, but to convert this to a conservation law, we need $\mathfrak{J}^\perp = 0$ on the boundaries, i.e., Neumann boundary conditions. Momentum is indeed conserved when the string endpoints move freely in the x direction. With Dirichlet conditions at both endpoints, the conserved charge, $\int d\sigma^1 \mathfrak{W}^0 = \int d\sigma^1 \mathfrak{J}^1$, is the minimal length of the fundamental string stretching between two localized D-branes. For a string with one free and one fixed endpoint, as in Fig. 3, none of the two charges is conserved. Two D-branes, one localized and one extending along the direction x , force indeed both the momentum and the stretching of the F-string along x to vanish.

现在考虑开世界面。对于可收缩围道，满足 $\oint d\sigma^\alpha \varepsilon_{\alpha\beta}\mathfrak{J}^\beta = 0$ ，但要由此得到守恒律，我们需要边界上满足 $\mathfrak{J}^\perp = 0$ ，也就是诺依曼边界条件。当弦端点在 x 方向自由运动时，动量确实是守恒的。若两端都取狄利克雷条件，守恒荷 $\int d\sigma^1 \mathfrak{W}^0 = \int d\sigma^1 \mathfrak{J}^1$ 是伸展在两个局域 D 膜之间的基本弦的最小长度。对于如图 3 所示一端自由一端固定的弦，两个荷都不守恒。一个局域、一个沿 x 方向伸展的两个 D 膜，确实会使 F 弦沿 x 方向的动量和伸展量都归零。

The discussion can be readily extended to spacetime supersymmetry. The corresponding conserved currents Q_α and $\varepsilon_{\alpha\beta}Q^\beta$, or the right- and left-moving combinations $(Q_R, 0)$ and $(0, Q_L)$, carry a $10d$ Majorana-Weyl spinor index that we suppress. An unpleasant feature of the NSR formalism is that spacetime supersymmetry is not manifest because the explicit expression of these currents is somewhat involved. All that is needed, however, for our purposes, here is that Q_R is constructed from the bosonized fermions ψ_R^μ and Q_L from the bosonized ψ_L^μ . Each complex chiral fermion $\psi_R^1 + i\psi_R^2$ corresponds to a free chiral boson $i\psi_R^1\psi_R^2 = \partial_-\phi_R$ which contributes a factor $\exp\left(\pm\frac{i}{2}\phi_R\right)$ to Q_R and likewise for left movers. We refer the reader to [8, 27] for details.

上述讨论可以很自然地推广到时空超对称。对应的守恒流 Q_α 和 $\varepsilon_{\alpha\beta}Q^\beta$ ，或者说左行和右行组合 $(Q_R, 0)$ 和 $(0, Q_L)$ ，带有一个我们省略不写的 $10d$ 马约拉纳-外尔旋量指标。NSR 形式体系一个不理想的特点是时空超对称并不显明，因为这些流的显式表达式相当复杂。但就我们此处的目的而言，只需要知道 Q_R 由玻色化费米子 ψ_R^μ 和 Q_L 构造而来，而 ψ_L^μ 也来自玻色化。每个复手征费米子 $\psi_R^1 + i\psi_R^2$ 对应一个自由手征玻色子 $i\psi_R^1\psi_R^2 = \partial_-\phi_R$ ，它会给 Q_R 贡献一个因子 $\exp\left(\pm\frac{i}{2}\phi_R\right)$，左动者同理。详情请读者参见 [8, 27] 。

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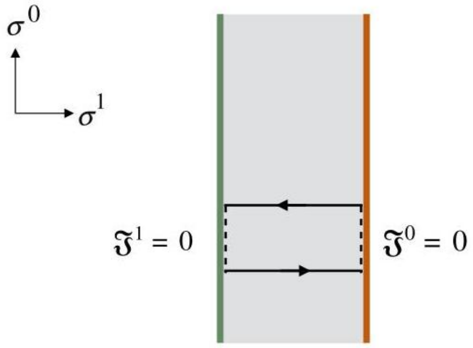
[9] The field strength of a 4-form gauge potential is a 5-form and the self-duality condition reads ${}^*F_{\mu_1\cdots\mu_5} = \frac{1}{5!}\varepsilon_{\mu_1\cdots\mu_{10}}F^{\mu_6\cdots\mu_{10}}$. With Lorentzian signature ${}^*({}^*F) = (-)^{(d-2)/2}F$, so real self-dual form fields only exist in 2 (mod 4) spacetime dimensions.

[9] 4-形式规范势的场强是 5-形式，自对偶条件可写为 ${}^*F_{\mu_1\cdots\mu_5} = \frac{1}{5!}\varepsilon_{\mu_1\cdots\mu_{10}}F^{\mu_6\cdots\mu_{10}}$ 。对于洛伦兹号差 ${}^*({}^*F) = (-)^{(d-2)/2}F$ ，实自对偶形式场仅存在于模 4 余 2 的时空维度中。

Fig. 3 For a coordinate with mixed (ND) boundary conditions neither of the two contour integrals vanishes on both boundaries of the open-string worldsheet

图 3 对于具有混合 (ND) 边界条件的坐标，两个围道积分都不会在开弦世界面的两个边界上都为零



On a closed worldsheet, the conserved charges $\oint Q_R$ and $\oint Q_L$ are independent. They generate the $\mathcal{N} = 32$ supersymmetries of the type-II supergravity theories. Consider next an open string with Neumann conditions for all X^μ . As explained in section "Neveu-Schwarz and Ramond Sectors," their fermionic partners obey the boundary conditions $\psi_R^\mu = \psi_L^\mu$. [10] The bosonization formula then shows that, before the GSO projections, $Q_R = Q_L$ on the boundary, and so $\int d\sigma^1 (Q_R - Q_L)$ is a candidate for a conserved charge. To survive, however, the GSO projections Q_L and Q_R must have the same chirality which is the case for the type-IIB superstring. We conclude that supersymmetric space-filling D9 branes can exist only in the type-IIB string theory.

在闭世界面上，守恒荷 $\oint Q_R$ 和 $\oint Q_L$ 是相互独立的。它们生成 II 型超引力理论的 $\mathcal{N} = 32$ 超对称。接下来考虑所有坐标都满足诺伊曼条件的开弦 X^μ 。正如“内沃-施瓦茨 sector 与拉蒙德 sector”一节所述，它们的费米子伙伴满足边界条件 $\psi_R^\mu = \psi_L^\mu$. [10] 玻色化公式表明，在 GSO 投影之前，边界上满足 $Q_R = Q_L$ ，因此 $\int d\sigma^1 (Q_R - Q_L)$ 是守恒荷的一个候选。但要能保留下来，GSO 投影 Q_L 和 Q_R 必须具有相同手性，这正是 IIB 型超弦的情况。我们因此得出结论: 超对称的填充满空间 D9 膜仅能存在于 IIB 型弦论中。

What about Dp branes with other values of p ? If X^j is a transverse coordinate obeying the Dirichlet condition $\partial_- X^j = -\partial_+ X^j$, then $\psi_R^j = -\psi_L^j$ on the boundary. Thus, Q_R is now identified with $(-)^{F_j} Q_L$ where $(-)^{F_j}$ anticommutes with ψ_L^j and commutes with all the other fermions. Acting on spinors, this is the operator $\Gamma\Gamma^j$. The supersymmetries left unbroken by a Dp brane are therefore in correspondence with those conserved charges:

那其他 p 取值的 Dp 膜呢? 如果 X^j 是满足狄利克雷条件 $\partial_- X^j = -\partial_+ X^j$ 的横向坐标，那么边界上满足 $\psi_R^j = -\psi_L^j$ 。因此，Q_R 现在等同于 $(-)^{F_j} Q_L$ ，其中 $(-)^{F_j}$ 与 ψ_L^j 反对易，且与所有其他费米子对易。作用在旋量上时，这就是算符 $\Gamma\Gamma^j$ 。因此，Dp 膜未破缺的超对称与这些守恒荷一一对应:

$$\int_0^\pi d\sigma^1 (Q_R - \Gamma_\perp Q_L), \text{ with } \Gamma_\perp = \prod_{j\in\perp} \Gamma\Gamma^j, \tag{58}$$

that survive the GSO projections. Since $\Gamma_\perp$ flips (does not flip) the spinor chirality when p is even (odd), supersymmetric Dp branes only exist in type-IIB string theory for p odd and in type-IIA string theory for p even. [11] The number of supersymmetries left unbroken by such D-branes is $\mathcal{N} = 16$, i.e., half of the $\mathcal{N} = 32$ supersymmetries of the closed string theory in the bulk.

在 GSO 投影后保留下来的守恒荷。由于当 p 为偶 (奇) 数时，$\Gamma_\perp$ 会改变 (不改变) 旋量手性，因此超对称 Dp 膜仅当 p 为奇数时存在于 IIB 型弦论，当 p 为偶数时存在于 IIA 型弦论。[11] 这类 D 膜未破缺的超对称数目为 $\mathcal{N} = 16$ ，即体空间闭弦理论原有 $\mathcal{N} = 32$ 超对称的一半。

Composite D-branes break more supersymmetries. Consider, for example, a pair of orthogonal D-branes, i.e., such that all coordinates have (NN), (DD), (ND), or (DN) boundary conditions. The boundary conditions for supersymmetry currents are $Q_R = \Gamma_\perp Q_L|_{\sigma^1=0}$ and $Q_R = \Gamma'_\perp Q_L|_{\sigma^1=\pi}$, where $\Gamma_\perp$ and $\Gamma'_\perp$ are the operators defined in (58) for the two D-branes at the string endpoints. The unbroken supersymmetries correspond therefore to solutions of the spinor equation:

复合 D 膜会破缺更多超对称性。例如，考虑一对正交 D 膜，即其所有坐标均满足 (NN)、(DD)、(ND) 或 (DN) 边界条件。超对称流的边界条件为 $Q_R = \Gamma_\perp Q_L|_{\sigma^1=0}$ 和 $Q_R = \Gamma'_\perp Q_L|_{\sigma^1=\pi}$ ，其中 $\Gamma_\perp$ 和 $\Gamma'_\perp$ 是弦端点处两个 D 膜对应在 (58) 式中定义的算符。因此未破缺的超对称性对应如下旋量方程的解:

$$(\Gamma_\perp)^{-1} \Gamma'_\perp S = S. \tag{59}$$

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[10] Modulo an irrelevant overall sign.

> [10] 模去一个无关的整体符号。

[11] If we relax the requirement of supersymmetry, odd- p branes also exist in type IIA and even- p branes in type IIB. These branes are unstable unless one applies an extra orbifold or orientifold projectionion [28].

> [11] 如果我们放宽超对称性的要求，IIA 型弦论中也存在奇数- p 膜，IIB 型弦论中也存在偶数- p 膜。这些膜是不稳定的，除非施加额外的轨道面或定向面投影 [28]。

By doing the Dirac-matrix algebra, one finds that $\mathcal{N}$ depends on the number of mixed (DN) and (ND) dimensions. If this number is zero, then $\mathcal{N} = 16$, if it is four or eight $\mathcal{N} = 8$, and in all other cases $\mathcal{N} = 0$, i.e., spacetime supersymmetry is completely broken unless the number of (DN) or (ND) coordinate is 0 (mod(4)).

> 通过狄拉克矩阵代数计算可得，$\mathcal{N}$ 依赖于混合 (DN) 和 (ND) 维度的数量。若该数量为 0，则 $\mathcal{N} = 16$ ；若该数量为 4 或 8，则 $\mathcal{N} = 8$ ；其余所有情况均为 $\mathcal{N} = 0$ ，即除非 (DN) 或 (ND) 坐标的数量模 4 余 0，否则时空超对称性会被完全破缺。

The generalization to rotated branes is straightforward: One must conjugate $\Gamma_\perp$ and $\Gamma'_\perp$ in Eq. (59) with the corresponding rotation matrices in spinor space. The conditions for unbroken supersymmetry can be easily worked out; see [18].

> 推广到旋转膜是很直接的: 只需将 (59) 式中的 $\Gamma_\perp$ 和 $\Gamma'_\perp$ 用旋量空间中对应的旋转矩阵做共轭变换即可。未破缺超对称性的条件可以很容易地推导出，参见文献 [18]。

D-Branes as Solitons

D 膜作为孤子

Many relativistic field theories have classical localized solutions that describe nondissipative excitations, alias solitons. Familiar examples are the monopoles, cosmic strings, or domain walls of Grand Unified field theories. D-branes can be considered as solitonic excitations of closed string theory. To introduce some basic facts about solitons, we will use a theory that plays an important role in the study of D-branes also for other reasons. This is the $N = 4$ supersymmetric Yang-Mills (SYM) theory in four dimensions [29] (for a review, see [30]).

> 许多相对论场论都存在描述非耗散激发的经典定域解，也就是孤子。大统一场论中我们熟悉的例子有磁单极、宇宙弦或畴壁。D 膜可以被看作闭弦理论的孤子类激发。为了介绍孤子的一些基本性质，我们会用到一个在 D 膜研究中因其他原因也十分重要的理论，这就是四维下的 $N = 4$ 超对称杨-米尔斯 (SYM) 理论 [29](综述参见 [30])。

$N = 4$ Super Yang-Mills

$N = 4$ 超杨-米尔斯

A convenient starting point is the ten-dimensional SYM:

一个方便的切入点是十维超杨-米尔斯理论:

$$S_{10}^{\text{YM}} = -\frac{1}{2g^2}\int d^{10}x\,\text{tr}\left(F_{\mu\nu}F^{\mu\nu} + 2i\bar{\lambda}\not{D}\lambda\right), \tag{60}$$

where λ is a Weyl-Majorana fermion in the adjoint representation of the gauge group $SU(n)$. The action (60) is invariant under the supersymmetry transformations $\delta A_\mu = -i\bar{\varepsilon}\Gamma_\mu\lambda$ and $\delta\lambda = \frac{1}{2}\Gamma_{\mu\nu}F^{\mu\nu}\varepsilon$, with ε a $10d$ Weyl-Majorana spinor.

其中 λ 是伴随表示下的外尔-马约拉纳费米子，对应规范群为 $SU(n)$ 。作用量 (60) 在超对称变换 $\delta A_\mu = -i\bar{\varepsilon}\Gamma_\mu\lambda$ 与 $\delta\lambda = \frac{1}{2}\Gamma_{\mu\nu}F^{\mu\nu}\varepsilon$ 下不变，其中 ε 是一个 $10d$ 外尔-马约拉纳旋量。

Upon reduction to $p+1$ dimensions, the $SO(1,9)$ Lorentz symmetry breaks to $SO(1,p)\times SO(9-p)$, where $SO(9-p)$ is called the R-symmetry. The gauge field decomposes into a $(p+1)$ -dimensional gauge field $A^{\mu=0,\dots,p}$ and $9-p$ scalars $A^{p+1,\dots,9} \equiv Y^{1,\dots,9-p}$. The gaugino λ reduces to a number of $(p+1)$ -dimensional gaugini transforming as a spinor of the R-symmetry group. [12] The scalars Y^j have a potential $V = -\sum_{i,j}\text{tr}\left(\left[Y^i, Y^j\right]^2\right)/2g^2$ descending from the commutator term in $F_{\mu\nu}$ which vanishes when the vacuum expectation values $\langle Y^j\rangle$ are mutually commuting. At a generic point of this so-called Coulomb branch, the unbroken gauge symmetry is abelian, $SU(n)\to U(1)^{n-1}$.

维约化到 $p+1$ 维后，$SO(1,9)$ 洛伦兹对称性破缺为 $SO(1,p)\times SO(9-p)$ ，其中 $SO(9-p)$ 被称为 R 对称性。规范场分解为一个 $(p+1)$ 维规范场 $A^{\mu=0,\dots,p}$ 和 $9-p$ 个标量场 $A^{p+1,\dots,9} \equiv Y^{1,\dots,9-p}$ 。戈迪诺 λ 约化为若干 $(p+1)$ 维戈迪诺，它们按 R 对称群的旋量表示变换。[12] 标量场 Y^j 拥有一个源自 $F_{\mu\nu}$ 中对易子项的势 $V = -\sum_{i,j}\text{tr}\left(\left[Y^i, Y^j\right]^2\right)/2g^2$ ，当真空期望值 $\langle Y^j\rangle$ 互相对易时该势为零。在这个所谓库仑分支的一般点上，未破缺的规范对称性是阿贝尔的，即 $SU(n)\to U(1)^{n-1}$ 。

This maximal SYM theory has some remarkable properties in four dimensions. First, it is conformal because the coupling g has vanishing beta function. Second, the coupling can be complexified by adding to the action a topological term:

这个最大超对称杨-米尔斯理论在四维空间具有一些引人注目的性质。首先，它是共形的，因为耦合常数 g 的 β 函数为零。其次，我们可以通过给作用量添加拓扑项将耦合常数复化为:

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} \quad \text{with} \quad \Delta S = -\frac{\theta}{32\pi^2}\int d^4x\,\text{tr}\left(F_{\mu\nu}{}^*F^{\mu\nu}\right). \tag{61}$$

Here, ${}^*F$ is the dual field strength, and ΔS is proportional to the Pontryagin index or instanton number of the gauge field. Last but not least, $N = 4$SYM is conjectured to be invariant [31] under the $\text{SL}(2,\mathbb{Z})$ duality transformations:

此处 $^*F$ 是对偶场强，ΔS 正比于庞特里亚金指标，即规范场的瞬子数。最后同样重要的是，人们猜想 $N=4$SYM 在 $\mathrm{SL}(2,\mathbb{Z})$ 对偶变换下不变 [31]:

$$\tau \to \frac{a\tau+b}{c\tau+d} \tag{62}$$

where a,b,c,d are integers with $ad-bc=1$.

其中 a,b,c,d 是满足条件 $ad-bc=1$ 的整数。

Take the gauge group $SU(2)$. By an $SO(6)_R$ rotation, we can bring any point on the Coulomb branch to $\langle Y_0^1\rangle = gv$, where $0,\pm$ label the neutral and charged components of the adjoint triplet under the unbroken $U(1)$. Charged fields get a mass $M_e = gv$. There exist also classical solutions, the ’t Hooft-Polyakov monopoles, which far from a smooth core look like a Dirac monopole of $U(1)$. Their mass $M_m = 4\pi v/g$ is mapped to that of the electric charges under the strong-weak duality $g\to 4\pi/g$. More generally, $\mathrm{SL}(2,\mathbb{Z})$ duality predicts that the $N=4$ SYM has dyonic excitations whose electric and magnetic charges Q_e, Q_m and mass M are

取规范群为 $SU(2)$。通过一次 $SO(6)_R$ 转动，我们可将库仑分支上的任意点变换为 $\langle Y_0^1\rangle = gv$ ，其中 $0,\pm$ 标记伴随三重态在未破缺 $U(1)$ 下的中性分量与带电分量。带电场获得质量 $M_e = gv$ 。此外还存在经典解——'t Hooft-Polyakov 磁单极，这类磁单极在远离光滑核心的区域表现为 $U(1)$ 的狄拉克磁单极。在强弱对偶 $g\to 4\pi/g$ 下，其质量 $M_m = 4\pi v/g$ 被映射为电荷的质量。更一般地，$\mathrm{SL}(2,\mathbb{Z})$ 对偶预言 $N=4$ 超杨-米尔斯存在双子激发，其电荷、磁荷 Q_e, Q_m 和质量 M 为

$$\begin{pmatrix} Q_e \\ -Q_m \end{pmatrix} = \sqrt{\frac{4\pi}{\mathrm{Im}\,\tau}} \begin{pmatrix} 1 - \mathrm{Re}\,\tau \\ 0\,\mathrm{Im}\,\tau \end{pmatrix} \begin{pmatrix} n_e \\ -n_m \end{pmatrix},\ M = v\sqrt{Q_e^2+Q_m^2}, \tag{63}$$

where $n_e, n_m \in \mathbb{Z}$ are relatively prime integers. Note that for $\theta = 2\pi\,\mathrm{Re}\,\tau \neq 0$ the magnetic monopole acquires an electric charge through Witten’s effect [32]. The dyon mass is invariant if the integer charges transform as an $\mathrm{SL}(2,\mathbb{Z})$ doublet:

其中 $n_e, n_m \in \mathbb{Z}$ 为互质整数。注意对于 $\theta = 2\pi\,\mathrm{Re}\,\tau \neq 0$ ，磁单极会通过威滕效应 [32] 获得电荷。若整数电荷按照 $\mathrm{SL}(2,\mathbb{Z})$ 二重态变换，双子质量保持不变:

$$\begin{pmatrix} n_e \\ -n_m \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} n_e \\ -n_m \end{pmatrix}. \tag{64}$$

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[12] In $p+1=6$ dimensions, the gaugini transform as a (pseudoreal) doublet of $SU(2)\subset SO(4)_R$, whereas in $p+1=4$ dimensions, they transform as a vector λ^a of $SU(4)\simeq SO(6)_R$.

[12] 在 $p+1=6$ 维中，gaugini(戈迪诺) 按照 $SU(2)\subset SO(4)_R$ 的 (赝实) 二重态变换，而在 $p+1=4$ 维中，它们按照 $SU(4)\simeq SO(6)_R$ 的向量表示 λ^a 变换。

Consider next the low-energy excitations in the monopole background. They include the massless neutral fields $\left(A^{\mu}, Y^{i}, \lambda^{a}\right)$ living in the bulk and the collective coordinates $\mathbf{X}(t)$ that describe the slow motion of the heavy monopole in space. The latter are the Goldstone bosons of broken translation symmetry. Since the monopole breaks also half of the $\mathcal{N}=16$ supersymmetries, there are in addition localized fermionic zero modes which provide the full spin content of a vector multiplet as expected from duality.

接下来考虑磁单极背景下的低能激发。这些激发包括位于体空间的无质量中性场 $\left(A^{\mu}, Y^{i}, \lambda^{a}\right)$，以及描述重磁单极在空间中缓慢运动的集体坐标 $\mathbf{X}(t)$。后者是平移对称性破缺产生的戈德斯通玻色子。由于磁单极同时破缺了一半的 $\mathcal{N}=16$ 超对称，额外还存在局域费米零模，如对偶所预言，这些零模给出了向量多重态的完整自旋内容。

The effective low-energy action has two terms, $S=S_{\text{bulk}}+S_{\text{monopole}}$ with S_{bulk} the supersymmetric Maxwell action and S_{monopole} the point-particle action of the monopole. Keeping only the bosonic fields, these read

低能有效作用量包含两项，$S=S_{\text{bulk}}+S_{\text{monopole}}$，其中 S_{bulk} 是超对称麦克斯韦作用量，S_{monopole} 是磁单极的点粒子作用量。仅保留玻色场时，两项可写为

$$S=-\frac{1}{2} \int d^{4} x\left[\frac{1}{2} F_{\mu \nu} F^{\mu \nu}+\left(\partial_{\mu} \mathbf{Y}\right)^{2}\right]-\frac{4 \pi}{g} \int d t\left[|\mathbf{Y}| \sqrt{-\dot{X}^{\mu} \dot{X}_{\mu}}+\widetilde{A}_{\mu} \dot{X}^{\mu}\right]$$

(65)

where $\mathbf{Y}$ stands for $\left(Y_{0}^{1}, \cdots, Y_{0}^{6}\right), F_{\mu \nu}$ is the field strength of the unbroken $U(1)$, and $\widetilde{A}_{\mu}$ its magnetic potential. The bulk fields $\mathbf{Y}$ and $\widetilde{A}_{\mu}$ in the second term are evaluated on the particle worldline $X^{\mu}(t)=(t, \mathbf{X})$. They were rescaled in order to have a canonically normalized bulk action. Dots stand for t -derivatives.

其中 $\mathbf{Y}$ 代表 $\left(Y_{0}^{1}, \cdots, Y_{0}^{6}\right), F_{\mu \nu}$，是未破缺 $U(1)$ 的场强，$\widetilde{A}_{\mu}$ 是其磁势。第二项中的体场 $\mathbf{Y}$ 和 $\widetilde{A}_{\mu}$ 是在粒子世界线 $X^{\mu}(t)=(t, \mathbf{X})$ 上取值的。为得到标准归一化的体作用量，这些场已做缩放处理。点号代表 t 对世界线参数的导数。

By linearizing this action around the vacuum $\langle\mathbf{Y}\rangle=(v, 0 \cdots, 0)$, one can show that there is no force between two heavy monopoles at rest. Put differently, the Coulomb repulsion cancels precisely the Yukawa attraction mediated by the Higgs scalar Y^{1}. The absence of a static force persists in the complete theory. It is a consequence of the fact that the ’t Hooft-Polyakov monopoles leave $\mathcal{N}=8$ unbroken supersymmetries and saturate the Bogomolny-Prasad-Sommerfeld (BPS) bound, $M \geq Q_{m} v$.[13] There actually exists a moduli space of classical multicenter monopole solutions with a metric that gives the leading velocity-dependent force between slowly moving monopoles [23].

通过对真空 $\langle\mathbf{Y}\rangle=(v, 0 \cdots, 0)$ 附近的该作用量线性化，可以证明两个静止的重磁单极之间不存在作用力。换句话说，库仑斥力恰好抵消了希格斯标量场 Y^{1} 介导的汤川吸引力。在完整理论中仍然不存在静作用力。这是源于以下事实: 特霍夫特-波里亚科夫磁单极保留了未破缺的 $\mathcal{N}=8$ 超对称性，并满足博戈莫尔尼-普拉萨德-萨默菲尔德 (BPS) 界，$M \geq Q_{m} v$.[13] 实际上，经典多中心磁单极解存在一个模空间，其度量给出了缓慢运动磁单极之间领头的与速度相关的作用力 [23]。

Effective Actions

有效作用量

The idea is now to consider the D-branes as semiclassical solitons of closed string theory. Whereas the closed strings move freely in the bulk, the open strings describe collective coordinates of the D-brane on which they attach; see Fig. 4. The low-energy excitations are the massless states of both open and closed strings. We will see that things fall nicely into place from this perspective.

现在的思路是将 D 膜视为闭弦理论的半经典孤子。闭弦在体空间中自由运动，开弦则描述其所附着的 D 膜的集体坐标；参见图 4。低能激发是开弦和闭弦共同的零质量态。我们会看到，从这个视角出发一切都条理清晰。

In the example of the previous section the effective bulk theory at energies $E \ll gv$ is the $4d, N = 4$ supersymmetric Maxwell theory. The ’ t Hooft-Polyakov monopole on the other hand reduces to a point particle with magnetic charge. In the case at hand, the effective theory in the bulk at $E \ll \sqrt{T_F}$ is one of the type-

在上一节的例子中，能量为 $E \ll gv$ 时的有效体理论是 $4d, N = 4$ 超对称麦克斯韦理论。另一方面，’t Hooft-Polyakov 磁单极约化为一个带磁荷的点粒子。对于我们当前讨论的情形，$E \ll \sqrt{T_F}$ 能下的体有效理论是某类 II 型-

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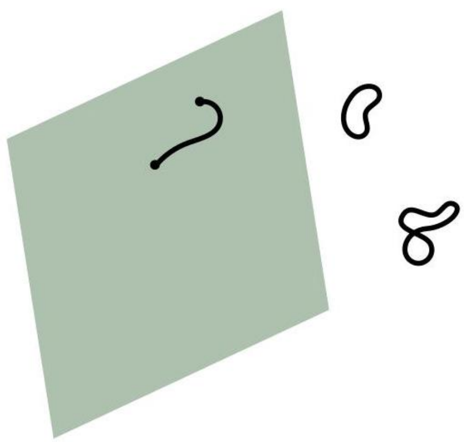
[13] The terms ”supersymmetric” and ”BPS” are used interchangeably in the literature though they are not, strictly speaking, equivalent.

[13] 尽管严格来说二者并不等价，但文献中“超对称”和“BPS”是互换使用的。

Fig. 4 The D-brane degrees of freedom are open strings that can split or join with closed strings in the bulk

图 4 D 膜的自由度是开弦，开弦可以在体空间中与闭弦分裂或结合



II supergravities in ten dimensions. The D-brane degrees of freedom on the other hand are described by a theory in $(p+1)$ dimensions: quantum mechanics for the D-particle, a two-dimensional sigma model for the D-string, or a four-dimensional field theory for the D3-brane. At the linear level, all these theories turn out to be our good old friend, the supersymmetric Maxwell theory reduced from 10 to $(p+1)$ dimensions.

十维中的 II 超引力。另一方面，D 膜的自由度由 $(p+1)$ 维理论描述:D 粒子对应量子力学，D 弦对应二维 sigma 模型，D3 膜对应四维场论。在线性层面，所有这些理论都正是我们熟悉的、从 10 维约化到 $(p+1)$ 维的超对称麦克斯韦理论。

This can be seen with the methods developed in section ”Superstrings.” The spectrum of an open superstring with only (NN) or (DD) coordinates is basically isomorphic to the left- or right-moving sector of the closed superstring. In particular, the massless states of an open string living on a Dp brane are

这可以用“超弦”一节中发展的方法证明。仅具有 (NN) 或 (DD) 坐标的开超弦的谱，基本同构于闭超弦的左行或右行扇区。特别地，位于 Dp 膜上的开弦的零质量态为

$$\psi_{-1/2}^{\mu=0,\ldots,9}\,|0\rangle_{\mathrm{NS}} \;\text{ and }\; |0\rangle_{\mathrm{R}}, \tag{66}$$

corresponding to a ten-dimensional vector field A^μ and a Weyl-Majorana fermion λ . These fields are reduced to $(p+1)$ dimensions, as described below Eq. (60), because the open string has no center-of-mass momentum in the (DD) directions transverse to the brane.

对应十维矢量场 A^μ 和外尔-马约拉纳费米子 λ。由于开弦在垂直于膜的 (DD) 方向上没有质心动量，这些场被约化到 $(p+1)$ 维，相关说明见式 (60) 下方。

The scalars Y^i are precisely the Goldstone bosons of broken Poincaré symmetry and the gaugini λ the Goldstini of broken supersymmetries. As for the gauge field A^μ , it is the Goldstone boson of the broken topological symmetry whose conserved charge is the closed-string winding number. Indeed, a closed string can break up on a D-brane wrapped around a compact dimension and undo its winding. Thus, the entire supermultiplet $\left(A^\mu, Y^i, \lambda^a\right)$ is a localized Goldstone supermultiplet whose existence could be predicted from symmetries alone.

标量场 Y^i 正是庞加莱对称性破缺产生的戈德斯通玻色子，引力微子 λ 是超对称性破缺产生的戈德斯提诺。对于规范场 A^μ , 它是破缺的拓扑对称性的戈德斯通玻色子, 该对称性的守恒荷是闭弦的绕数。事实上, 闭弦可以在缠绕紧致维的 D 膜上分裂, 解开它的缠绕。因此, 整个超多重态 $\left(A^\mu, Y^i, \lambda^a\right)$ 是局域化的戈德斯通超多重态，其存在仅通过对称性就可以预言。

Let us consider now the effective theory in more detail. We begin with type-IIA supergravity that can be more succinctly described by lifting to eleven dimensions. The bosonic part of the $11d$ action reads [24]

现在我们来更细致地讨论有效理论。我们从 IIA 超引力开始，它可以通过提升到十一维得到更简洁的描述。$11d$ 作用量的玻色子部分为 [24]

$$S_{11} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\left[\sqrt{-G}\left(R-\frac{1}{2}|F_4|^2\right)-\frac{1}{6}A_3\wedge F_4\wedge F_4\right] \tag{67}$$

with A_3 a 3-form gauge potential, $F_4 = dA_3$ its 4-form field strength, G_{MN} the metric, and κ_{11} the gravitational coupling. The normalization of $|F|^2$ is such that each component squared appears with unit coefficient. When no confusion is possible, subscripts will henceforth indicate the rank of a form. The Chern-Simons term at the end is the integral of an 11-form, and $\wedge$ stands for wedge product.

其中 A_3 是 3-形式规范势，$F_4 = dA_3$ 是它的 4-形式场强，G_{MN} 是度规，κ_{11} 是引力耦合。$|F|^2$ 的归一化满足每个分量平方都以单位系数出现。为了方便，此后我们用下标表示形式的阶数，不会造成歧义。末尾的陈-西蒙斯项是一个 11-形式的积分，$\wedge$ 代表外积。

Reduction on the circle $x^{10} = x^{10} + 2\pi$ gives the type-IIA action. The natural fields on the string worldsheet (the ”string-frame” fields) are given by the following combinations:

在圆周 $x^{10} = x^{10} + 2\pi$ 上约化得到 IIA 作用量。弦世界面上的自然场 (“弦框架”场) 由以下组合给出:

$$G_{MN}dx^M dx^N = e^{-2\Phi/3}G_{\mu\nu}dx^\mu dx^\nu + e^{4\Phi/3}\left[dx^{10} + e^{-\Phi}C_\mu' dx^\mu\right]^2$$

$$A_3 = B_2 \wedge dx^{10} + e^{-\Phi}C_3' \tag{68}$$

where $M = 0, \cdots, 10$ and $\mu = 0, \cdots, 9$. We can recognize in this decomposition the massless fields of type-IIA string theory; see Table 2: the NS-NS 2-form B_2 , the R-R forms C_1' and C_3' , and the dilaton Φ . The reason for the prime will be clear in a minute. Inserting (68) in S_{11} and dropping derivatives with respect to x^{10} gives the type-IIA supergravity action, $S_{\mathrm{IIA}} = S_{\mathrm{NS-NS}} + S_{\mathrm{RR}} + S_{\mathrm{CS}}$ with

其中 $M = 0, \cdots, 10$ 和 $\mu = 0, \cdots, 9$ 。我们可以从该分解中识别出 IIA 型弦论的无质量场；参见表 2:NS-NS 二形式 B_2 ，R-R 形式 C_1' 和 C_3' ，以及伸缩子 Φ 。加撇号的原因很快就会清楚。将 (68) 代入 S_{11} ，舍弃对 x^{10} 的导数后，得到 IIA 型超引力作用量，$S_{\mathrm{IIA}} = S_{\mathrm{NS-NS}} + S_{\mathrm{RR}} + S_{\mathrm{CS}}$ 为

$$S_{\mathrm{NSNS}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left(R + 4(d\Phi)^2 - \frac{1}{2}|dB_2|^2\right),$$

$$S_{\mathrm{RR}} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(\left|d\left(e^{-\Phi}C_1'\right)\right|^2 + \left|d\left(e^{-\Phi}C_3'\right) - e^{-\Phi}C_1' \wedge dB_2\right|^2\right),$$

$$S_{\mathrm{CS}} = -\frac{1}{4\kappa_{10}^2}\int B_2 \wedge d\left(e^{-\Phi}C_3'\right) \wedge d\left(e^{-\Phi}C_3'\right). \tag{69}$$

The merit of this clumsy rewriting is to exhibit an overall factor $e^{-2\Phi}$. The coupling of a dilaton background to the string is via a term $-\int \frac{d^2\sigma}{4\pi}\Phi\sqrt{-g}R$ in the worldsheet action. As explained in section ”Polyakov and Nambu-Goto Actions,” the factor $e^{-2\Phi}$ then shows that all terms in (69) are classical, i.e., come from closed-string sphere diagrams. Unless otherwise stated, the vacuum expectation value of Φ will be absorbed in κ_{10} , so that $\langle\Phi\rangle = 0$.

这种繁琐改写的优点是显示出整体因子 $e^{-2\Phi}$ 。伸缩子背景对弦的耦合是通过世界面作用量中的项 $-\int \frac{d^2\sigma}{4\pi}\Phi\sqrt{-g}R$ 实现的。正如“Polyakov 作用量与 Nambu-Goto 作用量”一节所述，因子 $e^{-2\Phi}$ 表明 (69) 中的所有项都是经典项，即来自闭弦球面图。除非另有说明，否则 Φ 的真空期望值会被吸收到 κ_{10} 中，因此 $\langle\Phi\rangle = 0$ 。

Now that we have exhibited the string-loop counting parameter, it makes more sense to eliminate the mixing of Φ with other fields by going to the ”Einstein frame”:

既然我们已经得到了弦圈计数参数，通过转到“爱因斯坦规范”消除 Φ 与其他场的混合就更合理了:

$$G_{\mu\nu} = e^{\Phi/2} g_{\mu\nu} \quad \text{and} \quad C_n' = e^{\Phi} C_n. \tag{70}$$

After this last field redefinition, the action (69) simplifies to

完成最后这次场重新定义后，作用量 (69) 简化为

$$\begin{aligned} S_{\mathrm{NSNS}} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2}(d\Phi)^2 - \frac{1}{2} e^{-\Phi} |dB_2|^2 \right), \\ S_{\mathrm{RR}} &= -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left(e^{-\Phi/2} |dC_1|^2 + e^{-3\Phi/2} |dC_3 - C_1 \wedge dB_2|^2 \right), \\ S_{\mathrm{CS}} &= -\frac{1}{4\kappa_{10}^2} \int B_2 \wedge dC_3 \wedge dC_3 \end{aligned} \tag{71}$$

This action is invariant under $10d$ diffeomorphisms and gauge transformations of the form fields. The transformation of the 1-form, which is a metric component in eleven dimensions, corresponds to reparametrizations $x^{10} \to x^{10} + \xi(x^{\mu})$. But as seen from Eq. (68), this also transforms the 3-form, so the gauge transformations of the RR fields are

该作用量在 $10d$ 微分同胚和形式场的规范变换下不变。十一维中作为度量分量的 1-形式变换对应于重新参数化 $x^{10} \to x^{10} + \xi(x^{\mu})$ 。但从式 (68) 可以看出，这也会变换 3-形式，因此 RR 场的规范变换为

$$\delta C_1 = d\xi \quad \text{and} \quad \delta C_3 = d\Lambda_2 + B_2 \wedge d\xi. \tag{72}$$

The field strength $F_4 = dC_3$ is therefore an exact but not gauge-invariant form, whereas the modified field strength $\widetilde{F}_4 = dC_3 - C_1 \wedge dB_2$ is gauge invariant but not exact since $d\widetilde{F}_4 = -dC_1 \wedge dB_2 \neq 0$. We take note of this subtlety but bypass it for the moment by setting $B_2 = 0$.

因此场强 $F_4 = dC_3$ 是恰当形式但不满足规范不变性，而修正后的场强 $\widetilde{F}_4 = dC_3 - C_1 \wedge dB_2$ 满足规范不变性但不是恰当形式，因为 $d\widetilde{F}_4 = -dC_1 \wedge dB_2 \neq 0$ 。我们注意到了这个微妙之处，但目前通过令 $B_2 = 0$ 绕过该问题。

Consider next the effective action of D-branes, starting with the more intuitive case of a D-particle. In the spirit of effective theories, one writes all possible terms consistent with symmetries plus any additional knowledge of the system. For a point particle, the two most relevant parameters are its mass T_{D0} and its charge ρ_{D0} , and the leading terms in the effective D-particle action are

接下来考虑 D 膜的有效作用量，从更直观的 D 粒子情况入手。按照有效理论的思路，我们写出所有满足对称性的可能项，再补充系统的额外已知信息。对于点粒子，两个最相关的参数是它的质量 T_{D0} 和电荷 ρ_{D0} ，有效 D 粒子作用量的领头项为

$$S_{\mathrm{D0}}=-T_{\mathrm{D0}}\int dse^{-\widehat{\Phi}}\sqrt{-\widehat{G}_{ss}}+\rho_{\mathrm{D0}}\int\widehat{C}_{1}. \tag{73}$$

The hats in this expression denote the pullback of the bulk supergravity fields to the worldline of the D-particle $Y^{\mu}(s)$, explicitly

本表达式中的帽子表示体超引力场拉回至 D 粒子的世界线 $Y^{\mu}(s)$ ，具体为

$$\widehat{\Phi}=\Phi(Y(s)),\ \widehat{G}_{ss}=G_{\mu\nu}(Y(s))\frac{dY^{\mu}}{ds}\frac{dY^{\nu}}{ds},\ \widehat{C}_{1}=C_{\mu}(Y(s))\frac{dY^{\mu}}{ds}ds. \tag{74}$$

A convenient parametrization is $Y^{0}(s)=s$; this is referred to in the jargon as the static gauge. The remaining D-brane coordinates $Y^{1},\cdots,Y^{9}$ are the scalars of the supersymmetric Maxwell multiplet reduced from 10 to 0+1 dimension, which is why we use the same symbol for them.

一种方便的参数化方式是 $Y^{0}(s)=s$ ；在行话中这被称为静态规范。剩余的 D 膜坐标 $Y^{1},\cdots,Y^{9}$ 是从 10 维约化到 0+1 维的超对称麦克斯韦多重态的标量，这也是我们对它们使用同一符号的原因。

The only thing not determined by symmetries in the action (73), besides T_{D0} and ρ_{D0} , is the coupling of the dilaton. This is fixed by the loop-counting argument used for the supergravity action. The tree-level interactions of closed and open strings come from worldsheets with the disk topology, and since $\chi_{\text{disk}}=1$, we expect a factor of $e^{-\Phi}$ in the string-frame action (note that the string-frame RR field is $e^{\Phi}C_{1}$) . The coupling of the dilaton in (73) is thus a string-theory input.

在作用量 (73) 中，除 T_{D0} 和 ρ_{D0} 外，唯一未被对称性确定的量是 dilaton 的耦合。这一点由超引力作用量所用的圈计数论证固定。闭弦和开弦的树级相互作用来自圆盘拓扑的世界面，由于 $\chi_{\text{disk}}=1$ ，我们预期弦论框架作用量中会出现一个因子 $e^{-\Phi}$ (注意弦论框架的 RR 场是 $e^{\Phi}C_{1}$))。因此 (73) 中 dilaton 的耦合是一个弦论输入项。

The extension to Dp branes with $p>0$ is straightforward. In terms of the tension $T_{\mathrm{D}p}$ and charge density $\rho_{\mathrm{D}p}$, the low-energy action reads

将其推广到带有 $p>0$ 的 Dp 膜是很直接的。用张力 $T_{\mathrm{D}p}$ 和荷密度 $\rho_{\mathrm{D}p}$ 表示，低能作用量为

$$S_{\mathrm{D}p}=-T_{\mathrm{D}p}\int\left[d^{p+1}s\right]e^{-\widehat{\Phi}}\sqrt{-\det\left(\widehat{\mathbf{G}}_{\alpha\beta}\right)}+\rho_{\mathrm{D}p}\int\widehat{C}_{p+1}, \tag{75}$$

where $\left(s^{0},\cdots,s^{p}\right)$ parametrize the brane worldvolume $Y^{\mu}(s^{\alpha})$, the pullback of the metric is $\widehat{G}_{\alpha\beta}=G_{\mu\nu}(Y)\partial_{\alpha}Y^{\mu}\partial_{\beta}Y^{\nu}$, and there is a similar expression for the pullback of the antisymmetric tensor C_{p+1} . Similar to point particles which couple to 1-form gauge potentials, p -branes may couple minimally to $(p+1)$ -form gauge fields. Thus, D2 branes can couple to C_{3} , while D4 branes and D6 branes can couple to the dual forms C_{5} and C_{7} defined by

其中 $(s^0,\cdots,s^p)$ 参数化膜世界体积 $Y^\mu(s^\alpha)$ ，度量的拉回是 $\hat{G}_{\alpha\beta}=G_{\mu\nu}(Y)\partial_\alpha Y^\mu\partial_\beta Y^\nu$ ，反对称张量 C_{p+1} 的拉回也有类似表达式。类似点粒子耦合 1-形式规范势，p 膜可以最小耦合 $(p+1)$ -形式规范场。因此，D2 膜可以耦合到 C_3 ，而 D4 膜和 D6 膜可以耦合到由下式定义的对偶形式 C_5 和 C_7

$$\varepsilon_p dC_{p+1}=*\left(dC_{7-p}\right) \tag{76}$$

with ε_p a sign; see Eq. (57). What about the D8 brane of the type-IIA superstring? It may couple to a 9-form gauge field whose equation implies that the field strength $F_{10}=dC_9$ is everywhere constant except at the position of the D8 brane. This explains why we did not find C_9 among the on-shell string states of Table 2. The piecewise constant F_{10} is called Romans mass and parametrizes a deformation of type-IIA supergravity in ten dimensions [33].

其中 ε_p 是一个符号，参见式 (57)。那 IIA 型超弦的 D8 膜呢？它可以耦合到 9-形式规范场，该规范场的方程表明，除 D8 膜所在位置外，场强 $F_{10}=dC_9$ 处处为常数。这就解释了为什么我们没有在表 2 的在壳弦态中找到 C_9 。分段常数的 F_{10} 被称为罗马斯质量，它参数化了十维 IIA 超引力的一种形变 [33]。

The story for type-IIB is basically the same with few notable differences. First, the theory has D(−1) branes, i.e., Dirichlet conditions for all coordinates including X^0 . They are interpreted as spacetime instantons and Eq. (75) as their Euclidean action. Second, there are two kinds of string, the D-string and the fundamental or F-string. They are charged, respectively, under the RR and NS-NS 2-forms C_2 and B_2 . Third, because $dC_4 = *(dC_4)$, the D3-branes carry both electric and magnetic charge - they are dyons. Finally, type-IIB theory admits space-filling D9 branes which cannot exist however on their own but require an exotic object called "orientifold." I will not discuss the ensuing theory of non-oriented strings (called type-I theory) here. A comprehensive review is Ref. [34].

IIB 理论的情况基本相同，只有几个值得注意的区别。第一，该理论存在 D(−1) 膜，即包括 X^0 在内所有坐标都满足狄利克雷边界条件。它们被解释为时空瞬子，式 (75) 就是它们的欧几里得作用量。第二，该理论有两种弦:D 弦和基本弦 (即 F 弦)，它们分别带 RR 2-形式 C_2 和 NS-NS 2-形式 B_2 的电荷。第三，因为 $dC_4=*(dC_4)$，D3 膜同时带有电荷和磁荷——它们是狄奥尼粒子。最后，IIB 理论允许充满空间的 D9 膜，但 D9 膜无法独立存在，需要一种称为“orientifold(定向轨形)”的奇异对象。我在此不讨论由此产生的非定向弦理论 (称为 I 型理论)，全面综述可见文献 [34]。

It is useful for the sequel to rewrite the D-brane action in the Einstein frame. From Eqs. (70) and (75), we find

为了后续讨论，将 D 膜作用量改写为爱因斯坦框架是很有用的。由式 (70) 和 (75)，我们得到

$$S_{\mathrm{D}p}=-T_{\mathrm{D}p}\int\left[d^{p+1}s\right]e^{-(p-3)\hat{\Phi}/4}\sqrt{-\det\left(\hat{g}_{\alpha\beta}\right)}+\rho_{\mathrm{D}p}\int\hat{C}_{p+1}. \tag{77}$$

Note in particular that the D3 brane does not couple to the dilaton, a fact that plays a key role in the AdS/CFT correspondence.

尤其需要注意，D3 膜不耦合 dilaton，这一性质在 AdS/CFT 对应中发挥着关键作用。

D-Brane Tension and Charge

D 膜张力与荷

We will now extract the D-brane tension and charge by calculating the force between two identical D-branes. This is Polchinski's seminal result [1]. Later, we will see a quicker derivation using T-duality.

我们现在将通过计算两个相同 D 膜之间的作用力来推导 D 膜的张力和荷。这是 Polchinski 的开创性结果 [1]。之后我们会看到用 T 对偶推导的更简便方法。

The static force between two D-branes can be computed in the effective low-energy theory by treating the branes as external sources for the supergravity fields C_{p+1}, Φ , and $h_{\mu v} = g_{\mu v} - \eta_{\mu v}$. At leading order in κ_{10} , we keep only terms linear in these perturbations:

两个 D 膜之间的静力学作用力可以在低能有效理论中计算, 我们将膜视为超引力场 C_{p+1}, Φ 和 $h_{\mu v} = g_{\mu v} - \eta_{\mu v}$ 的外源。在 κ_{10} 的领头阶下，我们只保留这些微扰的线性项:

$$S_{\mathrm{D}p} = \int d^{10}x\left(T^{\mu v}h_{\mu v} + j_{\Phi}\Phi + j_C C_{01\cdots p}\right) + \text{ non linear.} \tag{78}$$

We use as before the static gauge which is convenient for a slowly moving nearly planar Dp brane:

和之前一样，我们采用静态规范，这对于缓慢运动的近平面 Dp 膜十分方便:

$$Y^{\mu} = \left(s^0, s^1, \cdots, s^p, Y^{p+1}(s), \cdots Y^9(s)\right). \tag{79}$$

Inserting in Eq. (77) and expanding around $\Phi = 0$ gives

代入式 (77) 并在 $\Phi = 0$ 附近展开得到

$$T^{\mu v} = \frac{1}{2}T_{\mathrm{D}p}\delta\left(\mathbf{x}^{\perp}\right) \times \begin{cases} \eta^{\mu v} & \text{for } \mu, v = 0, \cdots, p \\ 0 & \text{otherwise} \end{cases} \tag{80}$$

$$j_{\Phi} = -\frac{p-3}{4}T_{\mathrm{D}p}\delta\left(\mathbf{x}^{\perp}\right) \text{ and } j_C = \rho_{\mathrm{D}p}\delta\left(\mathbf{x}^{\perp}\right), \tag{81}$$

where $\mathbf{x}_{\perp} = \left(x^{p+1}, \cdots, x^9\right)$ are the coordinates in the normal directions, the Dp brane is located at $\mathbf{x}_{\perp} = 0$, and we have dropped terms involving $\mathbf{Y}_{\perp}$ which correspond to open-string excitations.

其中 $\mathbf{x}_{\perp} = \left(x^{p+1}, \cdots, x^9\right)$ 是法向坐标，Dp 膜位于 $\mathbf{x}_{\perp} = 0$ ，我们舍去了对应开弦激发的包含 $\mathbf{Y}_{\perp}$ 的项。

Consider now two parallel Dp branes at $\mathbf{x}_{\perp} = \mathbf{r}$ and $\mathbf{x}_{\perp} = \mathbf{r}'$. Their interaction energy is given at leading order by the exchange of a virtual graviton, dilaton, or RR gauge field with the result

现在考虑两个平行 Dp 膜分别位于 $\mathbf{x}_{\perp} = \mathbf{r}$ 和 $\mathbf{x}_{\perp} = \mathbf{r}'$ 。它们的相互作用能在领头阶由虚引力子、胀子或 RR 规范场的交换给出，结果为

$$E_{\text{int}}\,\mathrm{T}_{\text{int}} = -2\kappa_{10}^2 \int d^{10}x \int d^{10}x' \left[j_\Phi \Delta j'_\Phi - j_C \Delta j'_C + T_{\mu\nu}\Delta^{\mu\nu,\rho\sigma}T'_{\rho\sigma} \right], \tag{82}$$

where T_{int} is the total time of interaction, $\Delta(x,x')$ is the scalar propagator, and $\Delta^{\mu\nu,\rho\sigma}(x,x')$ the propagator of the graviton. The negative sign in the contribution of the antisymmetric RR field is due to the fact that the ”exchanged component” $C_{01\ldots p}$ is timelike. [14]

其中 T_{int} 是总相互作用时间，$\Delta(x,x')$ 是标量传播子，$\Delta^{\mu\nu,\rho\sigma}(x,x')$ 是引力子传播子。反对称 RR 场贡献中的负号源于“交换分量”$C_{01\ldots p}$ 是类时的。[14]

The massless propagators in $10d$ Minkowski spacetime read

$10d$ 闵氏时空中的无质量传播子为

$$\Delta(x,x') = \int \frac{d^{10}k}{(2\pi)^{10}} \frac{e^{ik_\mu (x-x')^\mu}}{k^2} \quad \text{and}$$

$$\Delta^{\mu\nu,\rho\sigma}(x,x') = \left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{1}{4}\eta^{\mu\nu}\eta^{\rho\sigma}\right)\Delta(x,x'), \tag{83}$$

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[14] This is why the usual Yukawa and Coulomb forces have opposite signs.

[14] 这就是为什么通常汤川力和库仑力符号相反。

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where for the graviton we used the standard de Donder gauge. Putting everything together gives after some straightforward algebra:

其中对引力子我们使用了标准德东规范。整理所有结果后，经过一些直接代数运算得到:

$$E_{\text{int}} = 2V_p\kappa_{10}^2 \left(\rho_{\mathrm{D}p}^2 - T_{\mathrm{D}p}^2\right)\Delta_\perp(|\mathbf{r}-\mathbf{r}'|), \tag{84}$$

where V_p is the volume of the Dp -branes which can be made finite by wrapping them on a torus, and

其中 V_p 是 Dp 膜的体积，可以将其缠绕在环面上使体积有限，且

$$\Delta_\perp(r) = \frac{\Gamma\left(\frac{1}{2}d_\perp - 1\right)}{4\pi^{d_\perp/2}}\left(\frac{1}{r}\right)^{7-p} \tag{85}$$

is the scalar propagator in the $d_\perp = 9-p$ transverse dimensions. It is worth noting that the graviton and dilaton exchanges combine to make the prefactor that multiplies $\Delta_\perp(r)$ the same for all values of p .

是 $d_\perp = 9-p$ 个横向维中的标量传播子。值得注意的是，引力子和胀子交换结合后，使得乘在 $\Delta_\perp(r)$ 前的 prefactor 对任意 p 都相同。

We will now repeat the calculation in string theory and compare. Since the bulk supergravity fields correspond to massless closed strings, their exchange is given by the cylinder diagram shown in Fig. 5. To compute the full diagram, one would in principle need to know how each closed-string state couples to the D-branes. But there is another way to view the cylinder, as a loop of an open string stretching between the D-branes. Since we know the boundary conditions on the open-string worldsheet, the diagram can be readily computed [15]

我们现在将在弦论中重复这个计算并做对比。由于体超引力场对应无质量闭弦，它们的交换由图 5 所示的圆柱图给出。原则上，要计算完整图形需要知道每个闭弦态如何与 D 膜耦合。但我们可以换一种角度看这个圆柱: 它是伸展在两个 D 膜之间的开弦的圈图。由于我们已知开弦世界面的边界条件，因此可以很方便地计算该图 [15]

Let us recall the calculation of vacuum energy in ordinary quantum field theory. For a scalar field in d flat spacetime dimensions, the one-loop vacuum energy is

我们先来回顾普通量子场论中真空能的计算。对于 d 维平坦时空中的标量场，单圈真空能为

$$E_0 \mathrm{T} = -\frac{1}{2} \log \det \left(-\partial^2 + M^2\right) = -\frac{1}{2} \int \frac{d^d x d^d k}{(2\pi)^d} \log\left(k^2 + M^2\right)$$
$$\Rightarrow \frac{E_0}{V} = -\frac{1}{2} \int_0^\infty \frac{dt}{t} \int \frac{d^d k}{(2\pi)^d} e^{-(k^2+M^2)t} = -\int_0^\infty \frac{dt}{2t} (4\pi t)^{-d/2} e^{-M^2 t}.$$

(86)

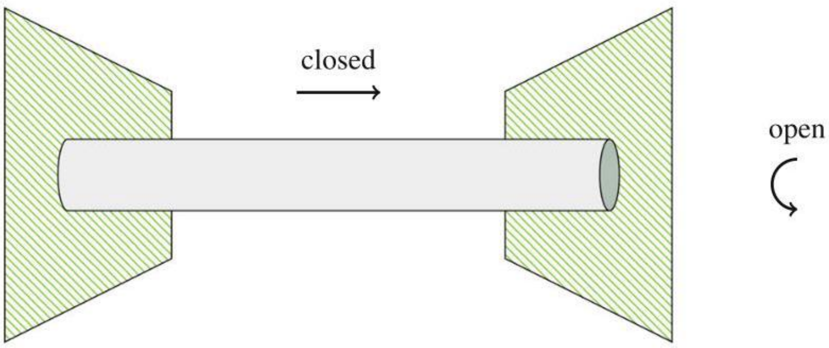


Fig. 5 A closed-string exchange or an open-string loop are given by the same cylinder diagram

图 5 闭弦交换和开弦圈图对应同一个圆柱图

---

[15] In the open-string channel, the cylinder diagram resembles a Casimir force, i.e., a force between probes that alter the fluctuations of the vacuum.

[15] 在开弦通道中，柱面图类似于卡西米尔力，也就是改变真空涨落的探针之间的作用力。

Here, $V\,\mathrm{T} = \int d^d x$ is the volume of spacetime, and in the lower line, we have expressed the logarithm using Schwinger's proper time. Repeating the calculation for particles with spin gives the same result times the number of spin states, while for fermions, the sign in front must be flipped.

此处，$V\,\mathrm{T} = \int d^d x$ 是时空的体积，我们在下划线中利用施温格真空中时表示了对数项。对自旋粒子重复计算会得到相同结果乘以自旋态的数量，而对于费米子，前方的符号必须翻转。

Treating now the stretched open string as a collection of point particles and using the above expression gives

现在我们将拉伸的开弦视为点粒子的集合，利用上述表达式可得

$$E_{\text{int}} = -V_p \int_0^\infty \frac{dt}{2t} \left(4\pi^2 \alpha' t\right)^{-(p+1)/2} \operatorname{Str}\left(e^{-\pi\alpha' M^2 t}\right), \tag{87}$$

where the supertrace (Str) stands for the sum over bosonic minus fermionic states of the string, and the momentum integration is over the $p+1$ dimensions of the D-brane worldvolume. We have also rescaled the integration variable $t \to \pi\alpha' t$ and define for convenience $q = \exp(-\pi t)$. Since all Neveu-Schwarz states are spacetime bosons while all Ramond states are fermions, the supertrace reads

其中-supertrace(Supertrace, Supertrace) 代表弦的玻色子态减去费米子态的和, 动量积分在 D 膜世界 volume 的 $p+1$ 维上进行。我们还重新标定了积分变量 $t \to \pi\alpha' t$ ，为方便起见定义了 $q = \exp(-\pi t)$ 。由于所有 Neveu-Schwarz 态都是时空玻色子，而所有 Ramond 态都是费米子，Supertrace 为

$$\operatorname{Str}\left(q^{\alpha' M^2}\right) = 2 \times \frac{1}{2} \operatorname{tr}_{\text{NS}}\left(1 + (-)^F\right) q^{\alpha' M^2} - 2 \times \frac{1}{2} \operatorname{tr}_{\text{R}}\left(1 + (-)^F\right) q^{\alpha' M^2}.$$

(88)

The insertions of the worldsheet-fermion parity operator $(-)^F$ implement the GSO projection, and the factor 2 accounts for the two orientations of the open string.

插入世界面费米子宇称算符 $(-)^F$ 实现 GSO 投影，因子 2 对应开弦的两种取向。

We have seen that the spectrum of an open string between identical parallel Dp branes is isomorphic to one chiral sector of the closed string. The mass formula, given in Eq. (16) for the bosonic string, is readily extended to the superstring with the result

我们已经看到，相同平行 Dp 膜之间开弦的能谱同构于闭弦的一个手征扇区。玻色弦的质量公式已在式 (16) 给出，它可以直接推广到超弦，结果为

$$\alpha' M^2 = \widehat{N} + \frac{|\mathbf{r} - \mathbf{r}'|^2}{4\pi^2 \alpha'} - \begin{cases} 1 & \text{for NS} \\ 0 & \text{for R} \end{cases} \tag{89}$$

where the level $\widehat{N}$ is the sum of oscillator frequency. The sum over states becomes then the canonical partition function of eight free bosons X^j and eight free fermions ψ^j . Partition functions for independent species factorize and one can compute them separately. Each boson contributes a factor $\eta(q)^{-1}$ where

其中能级 $\hat{N}$ 是振子频率之和。态求和于是成为 8 个自由玻色子 X^j 和 8 个自由费米子 ψ^j 的正则配分函数。独立种类的配分函数可以分解，我们可以分别计算它们。每个玻色子贡献一个因子 $\eta(q)^{-1}$ 其中

$$\eta(q) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \tag{90}$$

is the Dedekind eta function encountered in Eq. (24). For the fermions, the result depends on the sector and on whether $(-)^F$ is inserted or not in the trace. The four terms in Eq. (88) are

就是式 (24) 中遇到的戴德金 η 函数。对于费米子，结果取决于扇区，以及迹中是否插入了 $(-)^F$ 。式 (88) 中的四项分别是

1 

NS

$$= q^{-1/6} \prod_{n=0}^{\infty} (1 + q^{n+1/2})^8 \equiv (\theta_3/\eta)^4 ,$$

$(-)^F$ 

NS

$$= -q^{-1/6} \prod_{n=0}^{\infty} (1 - q^{n+1/2})^8 \equiv (\theta_4/\eta)^4 ,$$

1 

R

$$= 2^4 q^{1/3} \prod_{n=1}^{\infty} (1 + q^n)^8 \equiv (\theta_2/\eta)^4 ,$$

$(-)^F$ 

R

$$= (1 - 1)^4 q^{1/3} \prod_{n=1}^{\infty} (1 - q^n)^8 \equiv 0 \times (\theta_1/\eta)^4 .$$

(91a) (91b) (91c) (91d)

We have expressed the products in terms of the Jacobi theta functions $\theta_i(0 \mid q)$. Note that (91d) is zero because the unprojected Ramond sector has equal numbers of states with even and odd fermion parity. We kept this term for completeness since it contributes to other amplitudes. Collecting everything, we arrive at the following expression for the interaction energy Eq. (87):

> 我们已经用雅可比 θ 函数 $\theta_i(0 \mid q)$ 表示了乘积。注意 (91d) 等于零，因为未投影的拉蒙扇区中，奇偶费米子宇称的态数目相等。我们保留这一项是为了完整性，因为它对其他振幅有贡献。把所有结果汇总，我们得到相互作用能式 (87) 的如下表达式:

$$E_{\text{int}} = -2V_p \times \int_0^\infty \frac{dt}{2t}\left(4\pi^2\alpha' t\right)^{-(p+1)/2} e^{-|\mathbf{r}-\mathbf{r}'|^2 t/4\pi\alpha'} Z_{\text{open}}\left(q = e^{-\pi t}\right)$$

$$\text{with } Z_{\text{open}} = \frac{1}{2\eta^8}\left[\left(\frac{\theta_3}{\eta}\right)^4 - \left(\frac{\theta_4}{\eta}\right)^4 - \left(\frac{\theta_2}{\eta}\right)^4\right]. \tag{92}$$

The first thing to note about this expression is that it vanishes by Jacobi's abstruse identity $\theta_3^4 - \theta_4^4 - \theta_2^4 = 0$. Hence, there is no force between parallel identical D-branes at any separation $|\mathbf{r} - \mathbf{r}'|$. This is a consequence of unbroken supersymme-try, akin to the vanishing of the force between $N = 4$ SYM monopoles. Comparing in particular with the effective-theory result (84), we conclude that $\rho_{\mathrm{D}p} = T_{\mathrm{D}p}$, i.e., the tension of a D-brane is equal to its charge.

> 关于这个表达式首先要注意的是, 根据雅可比深奥恒等式 $\theta_3^4 - \theta_4^4 - \theta_2^4 = 0$ 它等于零。因此, 任意间距 $|\mathbf{r} - \mathbf{r}'|$ 下，平行全同 D 膜之间不存在作用力。这是未破缺超对称的结果，类似于 $N = 4$ 超对称杨-米尔斯理论中单极子之间作用力为零。特别和有效理论结果 (84) 对比后我们得出结论，$\rho_{\mathrm{D}p} = T_{\mathrm{D}p}$ 即 D 膜的张力等于它的电荷。

To find the actual value of the charge, we must isolate the exchange of the RR field C_{p+1} , in other words separate the NS-NS from the RR contributions in the closed-string channel of the cylinder diagram. For this, it is useful to consider the path-integral representation of the traces. As familiar from finite-temperature field theory, fermions are antiperiodic on the (Euclidean) time circle and periodic after insertion of $(-)^F$. [16] In Eq. (91), the open-string time runs upward, whereas the closed string propagates from left to right. We conclude that (91a, 91c) describe the exchange of a NS-NS closed string, whereas (91b, 91d) that of a closed string in the RR sector.

> 为了得到电荷的实际值，我们必须分离出 RR 场 C_{p+1} 的交换，也就是在柱面图的闭弦通道中分离 NS-NS 贡献和 RR 贡献。为此，考虑迹的路径积分表示会很有用。正如有限温度场论中我们熟知的，费米子在 (欧几里得) 时间圆上是反周期的，插入 $(-)^F$. [16] 后变为周期的。在式 (91) 中，开弦的时间向上延伸，而闭弦从左向右传播。我们可以得出结论:(91a, 91c) 描述 NS-NS 闭弦的交换，而 (91b, 91d) 描述 RR 扇区闭弦的交换。

In order to compare with effective field theory, the D-branes must be sufficiently distant, $|\mathbf{r} - \mathbf{r}'|^2 \gg \alpha'$, so that the contribution of massive closed-string states is exponentially small. In this limit, the integral in Eq. (92) is dominated by the region $t \ll 1$. This is the high-energy limit for the open string dominated by highly excited states. [17] To extract the leading behavior of the RR exchange (91b), we use the modular properties of the Jacobi θ -functions:

为了和有效场论对比，D 膜必须相距足够远，$|\mathbf{r}-\mathbf{r}'|^2 \gg \alpha'$，使得有质量闭弦态的贡献呈指数小。在该极限下，式 (92) 的积分由区域 $t \ll 1$ 主导。这是被高度激发态主导的开弦的高能极限。[17] 为了提取 RR 交换 (91b) 的领头行为，我们利用雅可比 θ 函数的模性质:

$$\left.\frac{\theta_4^4}{\eta^{12}}\right|_{q=e^{-\pi t}} = \left(\frac{t}{2}\right)^6 \left.\frac{\theta_2^4}{\eta^{12}}\right|_{q=e^{-4\pi/t}} \simeq \left(\frac{t}{2}\right)^6 2^4 + O\left(e^{-4\pi/t}\right). \tag{93}$$

Inserting in Eq. (92) gives the following contribution of the RR antisymmetric gauge field to the interaction energy per unit volume $\left(E_{\text{int}}/V_p\right)$:

代入式 (92)，得到 RR 反对称规范场对单位体积相互作用能的如下贡献 $\left(E_{\text{int}}/V_p\right)$:

$$\int_0^\infty \frac{dt}{2t}\frac{t^6}{4}\left(4\pi^2\alpha' t\right)^{-(p+1)/2} e^{-|\mathbf{r}-\mathbf{r}'|^2 t/4\pi\alpha'} = 2\pi\left(4\pi^2\alpha'\right)^{3-p}\Delta_\perp\left(|\mathbf{r}-\mathbf{r}'|\right).$$

Comparing with the supergravity result (84), we finally arrive at

与超引力结果 (84) 对比，我们最终得到

$$T_{\mathrm{D}p}^2 = \rho_{\mathrm{D}p}^2 = \frac{\pi}{\kappa_{10}^2}\left(4\pi^2\alpha'\right)^{3-p}. \tag{94}$$

---

[16] This follows from the Feynman-Kac formula and the Grassmann-integral identities:

[16] 这可由费曼-卡茨公式和格拉斯曼积分恒等式得到:

$$\operatorname{tr}(A) = \int d\bar\theta d\theta e^{-\bar\theta\theta}\langle -\bar\theta|A|\theta\rangle,\ \operatorname{tr}\left((-)^F A\right) = \int d\bar\theta d\theta e^{-\bar\theta\theta}\langle\bar\theta|A|\theta\rangle,$$

where $|\theta\rangle = |0\rangle - \theta\,|1\rangle$ and $\langle\bar\theta| = \langle 0| - \langle 1|\bar\theta$ are coherent fermionic bra and ket states and A any 2×2 -matrix operator acting on $|0\rangle$ and $|1\rangle$.

其中 $|\theta\rangle = |0\rangle - \theta\,|1\rangle$ and $\langle\bar\theta| = \langle 0| - \langle 1|\bar\theta$ 是相干费米子左矢和右矢，A 是任意作用在 $|0\rangle$ and $|1\rangle$ 上的 2×2 矩阵算符。

[17] Another way of stating the same thing is that the longer the stretching of the open string the softer its excitations.

[17] 另一种表述是: 开弦拉伸得越长，其激发越软。

This result is unusual when compared to ordinary solitons, i.e., smooth p -brane solutions of the nonlinear field equations. The tension of such solutions scales like m^{p+1}/g^2, where m is the typical mass of perturbative excitations and g is the strength of their interaction. For example, the mass of the 't Hooft-Polyakov monopole is $4\pi M_e/g^2$, with M_e the mass of the charged gauge bosons and g the Yang-Mills coupling constant;

see section " $N=4$ Super Yang-Mills." In string perturbation theory, the typical mass is $(\alpha')^{-1/2}$, while κ_{10} is related to the string coupling $g_s=\exp(\Phi_0)$ as follows [1]:

和普通孤子 (即非线性场方程的光滑 p 膜解) 相比，该结果很不寻常。这类解的张力标度为 m^{p+1}/g^2 ，其中 m 是微扰激发的典型质量，g 是其相互作用强度。例如，'t 霍夫特-波利亚科夫单极子的质量为 $4\pi M_e/g^2$ ，其中 M_e 是带电规范玻色子的质量，g 是杨-米尔斯耦合常数；参见章节"$N=4$ 超杨-米尔斯"。在弦微扰论中，典型质量为 $(\alpha')^{-1/2}$ ，而 κ_{10} 与弦耦合 $g_s=\exp(\Phi_0)$ 的关系如下 [1]:

$$\kappa_{10}^2=\frac{1}{2}(2\pi)^7\alpha'^4 g_s^2. \tag{95}$$

Inserting in (94) shows that $T_{\mathrm{D}p}$ scales like one inverse power of g_s . Thus, D-branes are heavier at weak coupling than fundamental strings, but lighter than conventional solitons such as the NS 5-brane that we will encounter in the coming section. This unusual $1/g_s$ scaling can be also related to the fact that the string-loop series expansion diverges much faster than in quantum field theory [4].

代入 (94) 可知，$T_{\mathrm{D}p}$ 的标度对应 g_s 的一次负幂次。因此，弱耦合下 D 膜比基本弦更重，但比我们下一节将介绍的常规孤子 (例如 NS5 膜) 更轻。这种不寻常的 $1/g_s$ 标度也和弦圈级数展开比量子场论发散快得多这一事实有关 [4]。

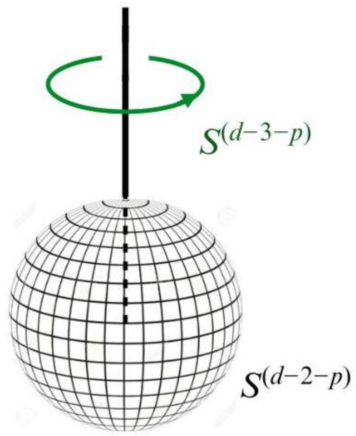
A nontrivial check of formula (94) is that it obeys Dirac's celebrated condition or rather its generalization to extended objects in higher dimension [35,36]. Recall that in $d=4$ electric and magnetic charges must obey $Q_eQ_m=2\pi n$ for integer n . This is the condition that the Dirac-string singularity of the monopole field cannot be detected by quantum interference of electric charges. Figure 6 illustrates the extension of the argument to higher dimensions. A p -dimensional magnetic object produces a field with a Dirac singularity that intersects the surrounding $(d-2-p)$ - sphere at the north pole. Electric charges are $(d-4-p)$ -dimensional branes which would pick a Bohm-Aharonov phase by wrapping a $(d-3-p)$ -sphere around the Dirac string. For $d=4$, both the electric and magnetic charges are point particles and the $(d-3-p)$ -sphere is a circle. For $d=10$, the dual charges are p -branes and $(6-p)$ -branes. In the conventions of the supergravity action (71), the canonically normalized gauge fields are $\sqrt{2}\kappa_{10}C_{p+1}$, and the quantization condition for the D-brane charges reads [18]

对公式 (94) 的一项重要验证是，它满足著名的狄拉克条件，更准确地说，是该条件向高维延展物体的推广 [35,36]。我们知道，在 $d=4$ 中，电和磁荷必须满足 $Q_eQ_m=2\pi n$ ，其中 n 为整数。该条件的物理意义是，单极场的狄拉克弦奇点无法通过电荷的量子干涉被探测到。图 6 展示了将该论证推广到高维的情况。p 维磁物体会产生带有狄拉克奇点的场，该奇点与周围的 $(d-2-p)$ 球面相交于北极。电荷是 $(d-4-p)$ 维膜，当 $(d-3-p)$ 球面环绕狄拉克弦时会获得玻姆-阿哈罗诺夫相位。对于 $d=4$ ，电和磁荷都是点粒子，此时 $(d-3-p)$ 球面是一个圆。对于 $d=10$ ，对偶荷分别是 p 膜和 $(6-p)$ 膜。在超引力作用量 (71) 的约定下，正则归一化的规范场是 $\sqrt{2}\kappa_{10}C_{p+1}$ ，D 膜电荷的量子化条件可写为 [18]

$$2\kappa_{10}^2\rho_{\mathrm{D}p}\rho_{\mathrm{D}(6-p)}=2\pi n \tag{96}$$

Fig. 6 Dirac's thought experiment for extended electric and magnetic objects in d dimensions

图 6 针对 d 维中延展电、磁物体的狄拉克思想实验



with n integer. The charges in (94) show that the cylinder calculation is consistent with this quantization condition.

其中 n 为整数。(94) 中的电荷表明，柱面计算结果与该量子化条件一致。

D-brane charges actually obey the minimal quantization, i.e., with $n = 1$. This should be contrasted to $N = 4$ SYM where the elementary dual charges $(1, 0)$ and $(0, 1)$ satisfy $Q_e Q_m = 2\pi n$ with $n = 2$; see Eq. (63). The explanation of this fact is simple: Fields transforming as doublets of the gauge group $SU(2)$ have 1/2 the charge of the adjoint triplets in the Coulomb phase. Even though such doublets are not part of the $N = 4$ supersymmetric theory, they can be coupled consistently and should thus be able to coexist with the 't Hooft Polyakov monopoles. In string theory, on the other hand, we just showed that fractional D-brane charges are not allowed. [19] This is in line with the lore that string theory does not admit couplings to other forms of matter or to external probes.

D 膜电荷实际上满足最小量子化，即对应 $n = 1$ 。这可以和 $N = 4$ 超对称杨-米尔曼理论对比，该理论中基本对偶荷 $(1, 0)$ 和 $(0, 1)$ 满足 $Q_e Q_m = 2\pi n$ ， 且 $n = 2$ ； 参见式 (63)。该事实的解释很简单: 在库仑相，属于规范群 $SU(2)$ 二重态表示的场，其电荷是伴随三重态电荷的 1/2 。尽管这类二重态不属于 $N = 4$ 超对称理论，但它们可以自洽地耦合，因此应当能够与't Hooft-Polyakov 单极共存。另一方面，弦论中我们刚刚证明分数 D 膜电荷是不被允许的。[19] 这符合弦论无法与其他形式物质或外部探针耦合的普遍结论。

Dualities

对偶性

Part of the initial excitement around D-branes came from the fact that they provided the charged excitations predicted by string dualities. This section gives a lightning account of dualities focussed on the role of D-branes.

D 膜最初引发轰动的部分原因在于，它提供了弦对偶理论所预言的带电激发态。本节将聚焦 D 膜的作用，扼要介绍对偶性。

The Power of T-Duality

T 对偶的能力

T-duality is a worldsheet symmetry and hence valid order by order in the string-loop expansion. It exchanges a free field X with another free field X' defined by $\partial_\alpha X = \varepsilon^{\alpha\beta}\partial_\beta X'$. Let X be a compact coordinate, $X = X + 2\pi r$ with r the radius of the circle. To simplify the formulae, we set $\alpha' = 1$, units will be restored when needed. The mode expansion of X reads

T 对偶是一种世界面对称，因此在弦圈展开中逐阶成立。它将自由场 X 与由 $\partial_\alpha X = \varepsilon^{\alpha\beta}\partial_\beta X'$ 定义的另一个自由场 X' 交换。设 X 为紧致坐标，$X = X + 2\pi r$ ，其中 r 是圆半径。为简化公式，我们设置 $\alpha' = 1$ ，需要时再恢复单位。X 的模式展开为

$$X = \frac{1}{2}\left(\frac{n}{r} - \tilde{n}r\right)\sigma^- + \frac{1}{2}\left(\frac{n}{r} + \tilde{n}r\right)\sigma^+ + \sum_{k\neq 0}\frac{i}{k\sqrt{2}}\left(a_k^\mu e^{-ik\sigma^-} + \tilde{a}_k^\mu e^{-ik\sigma^+}\right).$$

(97)

The difference with the earlier expression (6), besides the dummy summation index k , is in the zero-mode part of the above expansion. Because X is now compact, the center-of-mass momentum must be an integer multiple of $1/r$, and there is also an integer winding number $\tilde{n}$.

除哑求和指标 k 外，上述展开与之前的表达式 (6) 的区别在于零模部分。由于 X 现在是紧致的，质心动量必须是 $1/r$ 的整数倍，同时还存在整数绕数 $\tilde{n}$ 。

---

[18] The interpretation of this condition in the case $p = -1$ is somewhat different. It guarantees that the complex D-instanton action is single-valued in the backgroud of a D7 brane.

[18] 当 $p = -1$ 时，该条件的解释略有不同。它保证复 D 瞬子作用量在 D7 膜背景下是单值的。

[19] Except when localized on orbifolds, in which case Dirac’s argument does not apply [37].

[19] 轨道奇点局域的情况除外，此时狄拉克论证不成立 [37]。

From $\partial_0 X = \partial_1 X'$, one sees that T-duality interchanges momentum and winding. Hence, it is an exact equivalence between theories with radii r and $r' = 1/r$, provided one shifts also the dilaton by a constant:

从 $\partial_0 X = \partial_1 X'$ 可以看出，T 对偶交换动量和绕数。因此，只要同时将伸缩子平移一个常数，T 对偶就是半径为 r 和 $r' = 1/r$ 的理论之间的精确等价:

$$\Phi' = \Phi - \log r \tag{98}$$

The reason for this dilaton shift is that the interactions of string states with neither momentum nor winding in the compact dimension are controlled by the effective coupling in one lower dimension $\exp(\Phi_0)/\sqrt{2\pi r}$. This is invariant provided the dilaton transforms as above.

伸缩子发生平移的原因是，紧致维度中既无动量也无绕数的弦态的相互作用由低一维的有效耦合 $\exp(\Phi_0)/\sqrt{2\pi r}$ 控制。只要伸缩子按上述方式变换，该有效耦合就是不变的。

In the bosonic string theory, T-duality becomes an exact symmetry at the self-dual radius $r = 1$. The story for the superstrings is slightly twisted. To understand how, note that T-duality flips the sign of the right-moving fields:

在玻色弦理论中，T 对偶在自对偶半径 $r = 1$ 处成为精确对称性。超弦的情况略有不同。想要理解这一点，注意到 T 对偶会翻转右行场的符号:

$$\left(X'_R, \psi'_R\right) = (-X_R, -\psi_R)\,,\ \left(X'_L, \psi'_L\right) = (X_L, \psi_L)\,. \tag{99}$$

For X , this is a rewriting of $\partial_\alpha X = \epsilon^{\alpha\beta}\partial_\beta X'$, while its fermionic partner "goes along for the ride" in order to preserve worldsheet supersymmetry. Now, as was explained in section "D-Branes and Spacetime Supersymmetry," the operator that changes the sign of ψ_R^j is represented on the right-moving Ramond ground states by the matrix $\Gamma\Gamma^j$. Since this flips the chirality of the spinor, T-duality maps type-IIA theory to type-IIB and vice versa. It is thus an equivalence but not a symmetry even at $r = 1$.

对 X 而言，这只是 $\partial_\alpha X = \epsilon^{\alpha\beta}\partial_\beta X'$ 的重写，为了保持世界面对称性，其费米子伙伴也随之变换。正如“D 膜与时空超对称”一节所述，翻转 ψ_R^j 符号的算符在右行拉蒙基态上由矩阵 $\Gamma\Gamma^j$ 表示。由于这会翻转旋量的手征，T 对偶将 IIA 型理论映射为 IIB 型理论，反之亦然。因此即使在 $r = 1$ 处，它也只是等价关系而非对称性。

This can be also understood from the spacetime perspective. String theory compactified on a circle has a $U(1)_L \times U(1)_R$ gauge symmetry whose gauge fields are Kaluza-Klein components of the metric $G_{\mu\nu}$ and the Kalb-Ramond field $B_{\mu\nu}$. In the bosonic theory at the self-dual radius $r = 1$, this symmetry is enhanced to $SU(2)_L \times SU(2)_R$, thanks to the appearance of four extra massless vector bosons

这也可以从时空视角理解。圆紧致化的弦理论具有 $U(1)_L \times U(1)_R$ 规范对称性，其规范场是度规 $G_{\mu\nu}$ 和卡尔布-拉姆齐场 $B_{\mu\nu}$ 的卡鲁扎-克莱因分量。在自对偶半径 $r = 1$ 处的玻色弦理论中，由于四个额外无质量矢量玻色子的出现，该对称性增强为 $SU(2)_L \times SU(2)_R$

$$\alpha^\mu_{-1}\left|n = \tilde{n} = \pm 1\right\rangle \ \text{and}\ \ \tilde{\alpha}^\mu_{-1}\left|n = -\tilde{n} = \pm 1\right\rangle.$$

T-duality acts as a Weyl reflection of $SU(2)_R$ which is spontaneously broken at generic r by the expectation value of an (adjoint, adjoint) scalar. In the superstring theories, the above vector bosons are always massive and the $U(1)_L \times U(1)_R$ gauge symmetry is never enhanced.

T 对偶相当于 $SU(2)_R$ 的外尔反射，该对称性在一般 r 处会因 (伴随, 伴随) 标量的期望值而自发破缺。在超弦理论中，上述矢量玻色子始终是有质量的，$U(1)_L \times U(1)_R$ 规范对称性永远不会增强。

Let us next try to understand how T-duality acts on a D-brane. The first thing to note is that the transformation (99) swaps Neumann with Dirichlet boundary conditions. Thus, a $\mathrm{D}(p+1)$ brane wrapping the dimension x is transformed to a localized Dp brane in the T-dual theory and vice versa. This ties in nicely with the fact that the type-IIA and type-IIB theories have BPS Dp branes with p even, respectively, odd. Both T-dual objects look p -dimensional in the non-compact 9d spacetime. Asking that their tensions match gives

接下来我们来理解 T 对偶在 D 膜上的作用。首先需要注意的是，变换 (99) 交换了诺依曼边界条件和狄利克雷边界条件。因此，缠绕维度 x 的 $\mathrm{D}(p+1)$ 膜会在 T 对偶理论中变换为局域化的 Dp 膜，反之亦然。这恰好契合 IIA 型和 IIB 型理论分别存在 p 为偶数、奇数的 BPS Dp 膜这一事实。两个 T 对偶对象在非紧致 9d 时空中都是 p 维的。要求它们的张力匹配可得:

$$2\pi r T_{\mathrm{D}(p+1)} = T'_{\mathrm{D}p} = r T_{\mathrm{D}p}, \tag{100}$$

where in the second step we have used Eq. (98) together with the fact that the D-brane tensions scale like $\exp(-\Phi)$. The result of the cylinder calculation, Eq. (94), obeys this relation (after restoring the units $\alpha' = 1$) and is therefore consistent with T-duality, as expected.

第二步我们用到了式 (98)，以及 D 膜张力的标度关系为 $\exp(-\Phi)$ 。柱面计算的结果即式 (94) 满足该关系 (恢复单位 $\alpha' = 1$ 后)，因此和预期一致，与 T 对偶自洽。

Conversely, T-duality determines the tension of all Dp branes up to a common normalization. This can be fixed by imposing Dirac's minimal quantization, i.e., Eq. (96) with $n = 1$. The nontrivial content of the cylinder calculation is therefore to show that the D-branes form a complete set of RR charges.

反过来，T 对偶可以确定所有 Dp 膜的张力，只差一个公共归一化常数。这个归一化可以通过狄拉克最小量子化条件 (即代入 $n = 1$ 的式 (96)) 固定。因此柱面计算的核心意义在于证明 D 膜构成了完整的 RR 荷集合。

Let us take the discussion one step further and study the implications of T-duality for the D-brane action. [20] First, we extend the action of the fundamental string, Eq. (1), to account for nontrivial backgrounds of $G_{\mu\nu}, B_{\mu\nu}$ and of the gauge field A_μ that lives on the D-brane worldvolume:

我们进一步讨论，研究 T 对偶对 D 膜作用量的影响。[20] 首先，我们推广基本弦的作用量即式 (1)，以计入 $G_{\mu\nu}, B_{\mu\nu}$ 的非平凡背景，以及 D 膜世界体积上存在的规范场 A_μ：

$$S_F = -T_F \int_\Sigma d^2\sigma \sqrt{-\det\left(\widehat{G}_{\alpha\beta}\right)} + T_F \int_\Sigma \widehat{B}_2 + T_F \oint_{\partial\Sigma} \widehat{A}. \tag{101}$$

The first two terms are analogous to those of the D-brane action, Eq. (77). We have used the fact that the fundamental string (alias F1 brane) is a BPS charge for the Kalb-Ramond 2-form B_2 , with hats indicating the pullback on the worldsheet, e.g., $\widehat{B}_2 = \frac{1}{2} B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu d\sigma^\alpha \wedge d\sigma^\beta$, etc. The extra term $\oint_{\partial\Sigma} \widehat{A}$ shows that the endpoint of the string is charged under the Maxwell field of the corresponding D-brane. We normalized for convenience A_μ so that this charge is $\pm T_F$.

前两项对应 D 膜作用量式 (77) 的类似项。我们知道基本弦 (又称 F1 膜) 是 Kalb-Ramond 2 形式 B_2 的 BPS 荷，带帽量表示世界面上的拉回，例如 $\widehat{B}_2 = \frac{1}{2}B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu d\sigma^\alpha \wedge d\sigma^\beta$ 等等。额外项 $\oint_{\partial\Sigma}\widehat{A}$ 说明弦端点在对应 D 膜的麦克斯韦场下带荷。为方便起见我们归一化 A_μ，使得该荷为 $\pm T_F$。

Note that the last two terms in (101) are "topological" terms, i.e., they do not couple to the metric of a curved worldsheet. They can therefore be added to the Polyakov action Eq. (2) without introducing the Liouville field.

注意式 (101) 的最后两项是“拓扑”项，即它们不耦合弯曲世界面的度规。因此可以将它们添加到波利亚科夫作用量式 (2) 中，无需引入刘维尔场。

An immediate consequence of (101) is that the gauge invariance $\delta B_2 = d\omega_1$ of closed strings must be modified in the presence of D-branes as follows:

式 (101) 的一个直接结论是，闭弦的规范不变性 $\delta B_2 = d\omega_1$ 在 D 膜存在时必须修改为如下形式:

$$\delta B_2 = d\omega_1 \quad \text{and} \quad \delta A = -\omega_1|_{\mathrm{D}p}, \tag{102}$$

where $\omega_1|_{\mathrm{D}p}$ is the pullback of the 1-form gauge parameter to the D-brane [21] If the D-branes at the string endpoints are different, both worldvolume gauge fields transform as above modulo a sign for orientation.

其中 $\omega_1|_{\mathrm{D}p}$ 是 1 形式规范参数到 D 膜 [21] 的拉回。若弦端点所在的两个 D 膜不同，两个世界体积规范场都按上述规则变换，仅因取向差一个符号。

Consider now a Dp brane transverse to a compact dimension, say $x^9 = x^9 + 2\pi r$. We use the static gauge, Eq. (79), in which the embedding of the D-brane in the 9th dimension is given by the coordinate function $Y^9(s^\alpha)$ with $\alpha = 0, \cdots, p$. For simplicity, we set all other transverse fluctuations to zero. The pullback of the RR gauge field on the Dp-brane worldvolume then reads

现在考虑一个垂直于紧致维度 (例如 $x^9 = x^9 + 2\pi r$) 的 Dp 膜。我们采用静态规范即式 (79)，D 膜在第 9 维的嵌入由坐标函数 $Y^9(s^\alpha)$ 描述，满足 $\alpha = 0, \cdots, p$。为简化，我们令所有其他横向涨落为零。RR 规范场在 Dp 膜世界体积上的拉回为:

$$\widehat{C}_{p+1} = C_{\mu_0\cdots\mu_p} dY^{\mu_0} \wedge \cdots \wedge dY^{\mu_p} \quad \text{with} \quad dY^\mu = \left(\delta^\mu_\alpha + \delta^\mu_9 \partial_\alpha Y^9\right) ds^\alpha. \tag{103}$$

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[20] A review of duality constraints on D-brane actions is [38].

[20] 关于 D 膜作用量的对偶约束综述可见文献 [38]。

[21] We don't use here the hat notation to avoid confusion with pullbacks on the F-string worldsheet.

[21] 我们在此不使用带 hat 的记号，以避免与 F 弦世界面上的拉回映射混淆。

Let us try to understand how this transforms if the coordinate X^9 is T-dualized. First, as explained above, the Dp brane becomes a D $(p+1)$ brane wrapped around the x^9 circle. Second, $Y^9(s^\alpha)$ is traded for the component $A^9(s^\alpha)$ of the gauge field on the worldvolume of the D $(p+1)$ brane; see section "Effective Actions." Finally, the RR bispinor field $\mathbf{C}$ transforms to $\Gamma\Gamma^9\mathbf{C}$. Combining it all and working out the algebra leads to the following transformation of the RR form Eq. (103):

我们来理解一下，对坐标 X^9 做 T 对偶后，该对象会如何变换。首先，如上文所述，Dp 膜会变成缠绕 x^9 圆周的 D $(p+1)$ 膜。其次，$Y^9(s^\alpha)$ 被替换为 D $(p+1)$ 膜世界面上规范场的分量 $A^9(s^\alpha)$；参见“有效作用量”一节。最后，RR 双旋量场 $\mathbf{C}$ 变换为 $\Gamma\Gamma^9\mathbf{C}$ 。将所有结果结合并整理代数推导后，可得式 (103) 中 RR 形式的如下变换:

$$\widehat{C}_{p+1} \to \widehat{C}_{p+2} + \widehat{C}_p \wedge F \tag{104}$$

where $F = dA$. Since T-duality is an equivalence of theories, we have just learned that in addition to C_{p+2} a D $(p+1)$ brane couples also to C_p when its worldvolume gauge field is switched on [39, 40] .

其中 $F = dA$ 。由于 T 对偶是理论间的等价性，我们由此可知，当开启世界面规范场 [39, 40] 时，除 C_{p+2} 外，D $(p+1)$ 膜还会与 C_p 耦合。

Extending this reasoning to more dimensions and applying it also to the pullback of the metric in the Nambu-Goto part leads to the following generalization of the action of an isolated Dp brane [22] :

将该推理推广到更多维度，并应用于南布-戈登项中的度量拉回，可得到单个 Dp 膜作用量的如下推广形式 [22]：

$$S_{\mathrm{D}p} = -T_{\mathrm{D}p} \int [ds]\, e^{-\Phi} \sqrt{-\det\left(\widehat{G}_{\alpha\beta} + \widehat{B}_{\alpha\beta} + F_{\alpha\beta}\right)} + \rho_{\mathrm{D}p} \int \left[e^{\widehat{B}+F} \wedge \widehat{\mathbf{C}}\right]_{p+1}$$

(105)

where here $\mathbf{C} = \sum_n C_n$ is the formal sum of all RR forms, the exponential is likewise the formal finite sum $e^{\mathcal{F}} = 1 + \mathcal{F} + \frac{1}{2}\mathcal{F} \wedge \mathcal{F} + \cdots$, and $[\mathcal{A}]_{p+1}$ projects the $(p+1)$ -form out of the sum of forms $\mathcal{A}$. The coupling of $B_{\mu\nu}$ in this action has been fixed by invariance under the gauge transformations (102). The two terms in Eq. (105) are known as the Dirac-Born-Infeld (DBI) [41] and Wess-Zumino (WZ) terms of the D-brane action.

此处 $\mathbf{C} = \sum_n C_n$ 是所有 RR 形式的形式和，指数也同样是形式有限和 $e^{\mathcal{F}} = 1 + \mathcal{F} + \frac{1}{2}\mathcal{F} \wedge \mathcal{F} + \cdots$ ，$[\mathcal{A}]_{p+1}$ 从形式和 $\mathcal{A}$ 中投影出 $(p+1)$ -形式。该作用量中 $B_{\mu\nu}$ 的耦合已由规范变换 (102) 下的不变性固定。式 (105) 中的两项分别是 D 膜作用量中著名的狄拉克-玻恩-因费尔德 (DBI) 项 [41] 和韦斯-祖米诺 (WZ) 项。

The Wess-Zumino term raises a puzzle about the proper definition of RR charge [42, 43]. To illustrate the problem, consider the D-particle charge induced on a spherical D2-brane. The gauge-invariant charge $q = \rho_{\mathrm{D2}} \int_{S^2}\left(\widehat{B}_2 + F\right)$, the one that couples to the potential C_1 , is neither conserved nor quantized. Indeed, the charge of a D2 brane tracing a worldvolume W changes by an amount $\delta q = \int_W \widehat{H}_3$ which does not vanish in general unless $H_3 = dB_2 = 0$. To remedy the situation, one can define the alternative "Page charge":

韦斯-祖米诺项引出了 RR 电荷恰当定义的难题 [42,43]。为说明该问题，考虑球形 D2 膜上诱导出的 D 粒子电荷。耦合到势场 C_1 的规范不变电荷 $q=\rho_{\mathrm{D2}}\int_{S^2}\left(\widehat{B}_2+F\right)$ 既不守恒也不量子化。实际上，世界体积为 W 的 D2 膜，其电荷的改变量为 $\delta q=\int_W \widehat{H}_3$ ，除非满足 $H_3=dB_2=0$ ，否则该改变量一般不为零。为解决该问题，我们可以定义另一种“佩奇电荷”：

$$q_{\text{Page}}=\rho_{\mathrm{D2}}\int_{S^2} F. \tag{106}$$

This is indeed quantized, as follows from Dirac's quantization of the worldvolume flux $\int_{S^2}F=2\pi n/T_F$ [23] and from the tension formula Eq. (94) which implies that $\rho_{\mathrm{D2}}\int_{S^2}F=n\rho_{\mathrm{D0}}$. Something however has to give, and this is the invariance of the charge under large gauge transformations of B_2 , i.e., transformations δB_2 such that $d\left(\delta B_2\right)=0$ but $\delta B_2\neq d\Lambda_1$. Since $\delta F=-\delta B_2$, such large gauge transformations change q_{Page} . The nonexistence of an invariant quantized RR charge is a general phenomenon that can be traced to the anomalous Bianchi identities and the Chern-Simons terms of supergravity actions.

这确实是量子化的，可由狄拉克对世界体积通量 $\int_{S^2}F=2\pi n/T_F$ [23] 的量子化推导得出，也可由式 (94) 的张力公式推导得出，该公式暗示 $\rho_{\mathrm{D2}}\int_{S^2}F=n\rho_{\mathrm{D0}}$ 。但总有一处需要让步，这就是电荷在 B_2 的大规范变换下的不变性，即变换 δB_2 满足 $d\left(\delta B_2\right)=0$ 但 $\delta B_2\neq d\Lambda_1$ 。由于 $\delta F=-\delta B_2$ ，这类大规范变换会改变 q_{Page} 。不存在不变的量子化 RR 电荷是普遍现象，可追溯到反常比安基恒等式和超引力作用量的陈-西蒙斯项。

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[22] Our T-duality argument assumed that background fields do not depend on x^9 . This is because T-duality would trade this for dependence on a dual coordinate $\tilde{x}^9$ conjugate to winding rather than momentum, and geometric intuition would be lost. But after discovering the couplings required by T-duality, one can extend them to x^9 -dependent backgrounds by locality in the $r\gg 1$ limit.

[22] 我们的 T 对偶论证假设背景场不依赖于 x^9 。这是因为 T 对偶会将其替换为对绕数而非动量共轭的对偶坐标 $\tilde{x}^9$ 的依赖，几何直观就会丢失。但在发现 T 对偶要求的耦合后，我们可以通过 $r\gg 1$ 极限下的局域性将其推广到依赖 x^9 的背景。

[23] Recall that F has been normalized so that string endpoints carry charge T_F .

[23] 回想一下，F 已经归一化，使得弦端点携带电荷 T_F 。

We note for completeness, before moving on, that it is possible to extend the D-brane action (105) so as to include the worldvolume gaugini [44] and that both the WZ and the DBI terms receive curvature corrections [45,46]. These are useful in many applications but they are beyond our scope here.

在继续下一步之前，我们为完整性说明: 可以扩展 D 膜作用量 (105) 以纳入世界体积 gaugini[44]，且 WZ 项和 DBI 项都会受到曲率修正 [45,46]。这些修正对许多应用都很有用，但超出了本文的讨论范围。

SL(2, Z) Duality of Type IIB

IIB 型弦的 SL(2, Z) 对偶性

In section " $N = 4$ Super Yang-Mills," we gave a rudimentary account of the duality of $N = 4$ SYM theory. The effective Maxwell theory on the Coulomb branch has a continuous symmetry under $\mathrm{SL}(2,\mathbb{R})$ rotations of $F_{\mu\nu}$ and $-^*F_{\mu\nu}$. The discrete subgroup $\mathrm{SL}(2,\mathbb{Z})$ that respects the quantization of electric and magnetic charges is conjectured to generate exact equivalences of the full quantum theory [31]. It includes a nontrivial element called S-duality that maps strong to weak coupling, $\tau \to -1/\tau$. The role of supersymmetry is to protect both τ and the dyon spectrum from quantum corrections, making it possible to test the conjecture.

在“$N = 4$ 超杨-米尔斯”一节中，我们对 $N = 4$ SYM 理论的对偶性做了基础介绍。库仑分支上的有效麦克斯韦理论在 $\mathrm{SL}(2,\mathbb{R})$ 对 $F_{\mu\nu}$ 和 $-^*F_{\mu\nu}$ 的旋转下具有连续对称性。满足电、磁荷量子化条件的离散子群 $\mathrm{SL}(2,\mathbb{Z})$ 被猜想可生成全量子理论的精确等价性 [31]，其中包含一个非平凡元素，即 S 对偶，它可将强耦合映射为弱耦合，$\tau \to -1/\tau$ 。超对称的作用是保护 τ 和磁单子谱不受量子修正，从而使我们可以检验这个猜想。

The U-dualities of string theory [47-49] are similar in spirit but much richer. They are discrete remnants of the exceptional symmetry groups [50] of the maximal supergravities in various dimensions that are believed to relate consistent quantum-gravity theories. U-dualities include both T-dualities and strong-weak equivalences. Tests of the latter use various protected quantities, in particular the spectra of BPS excitations. For a comprehensive review, the reader may consult Ref. [51]; here, we only present a few salient features.

弦论的 U 对偶性 [47-49] 在本质上与之相似，但内容丰富得多。它们是各类维度下最大超引力例外对称群 [50] 的离散残余，被认为可以关联自洽的量子引力理论。U 对偶性同时包含 T 对偶和强弱耦合等价。检验后者需要用到各类受保护的量，尤其是 BPS 激发谱。读者可参阅文献 [51] 获得全面综述，本文仅介绍若干显著特征。

In $d = 9$ dimensions, the U-duality group is $\mathrm{SL}(2,\mathbb{Z}) \times \mathbb{Z}_2$. The $\mathbb{Z}_2$ factor is the T-duality of the previous subsection that identifies the type-IIA and type-IIB string theories. These two theories are perturbatively equivalent, but the action of $\mathrm{SL}(2,\mathbb{Z})$ on them is strikingly different.

在 $d = 9$ 维中，U 对偶群为 $\mathrm{SL}(2,\mathbb{Z}) \times \mathbb{Z}_2$ 。其中 $\mathbb{Z}_2$ 因子是上一小节介绍的 T 对偶，它将 IIA 型弦论与 IIB 型弦论视为等价。这两种理论微扰层面等价，但 $\mathrm{SL}(2,\mathbb{Z})$ 对二者的作用截然不同。

We start with type-IIB which is conceptually simpler. The effective supergravity has in addition to the metric $g_{\mu\nu}$, a complex scalar field $\tau = C + ie^{-\Phi}$, two 2-form fields B_2 and C_2 , the self-dual 4-form C_4 , and their fermionic partners. The action has a continuous $\mathrm{SL}(2,\mathbb{R})$ symmetry that leaves $g_{\mu\nu}$ and C_4 invariant and acts on the other fields as follows:

我们从概念上更简单的 IIB 型弦论开始介绍。有效超引力除度规 $g_{\mu\nu}$ 外，还包含一个复标量场 $\tau = C + ie^{-\Phi}$ 、两个 2 形式场 B_2 和 C_2 、自对偶 4 形式场 C_4 ，以及它们的费米子伙伴。作用量具有连续 $\mathrm{SL}(2,\mathbb{R})$ 对称性，该对称性保持 $g_{\mu\nu}$ 和 C_4 不变，对其余场的作用形式如下:

$$\begin{pmatrix} B_2' \\ -C_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} B_2 \\ -C_2 \end{pmatrix} \quad \text{and} \quad \tau' = \frac{a\tau + b}{c\tau + d}. \tag{107}$$

It is conjectured that $\mathrm{SL}(2,\mathbb{Z}) \subset \mathrm{SL}(2,\mathbb{R})$ is an exact duality of the quantum theory. Let us see how D-branes fit with this conjecture.

现有猜想认为 $\mathrm{SL}(2,\mathbb{Z}) \subset \mathrm{SL}(2,\mathbb{R})$ 是量子理论的精确对偶。我们来看 D 膜如何契合这个猜想。

Begin with the D-instanton whose tension reads $T_{\mathrm{D}(-1)} = 2\pi/g_s$, see Eqs. (94) and (95). Plugging in the action (75), rotating to imaginary time and reinserting $g_s = e^{\Phi_0}$ inside the dilaton gives the Euclidean action

首先从 D 瞬子开始，它的张力为 $T_{\mathrm{D}(-1)} = 2\pi/g_s$ ，见式 (94) 和 (95)。将其代入作用量 (75)，旋转到虚时间再将 $g_s = e^{\Phi_0}$ 重新代回 dilaton，可得欧几里得作用量

$$S_{\mathrm{D}(-1)} = 2\pi i\tau \tag{108}$$

This makes manifest that $\tau \to \tau+1$ is a symmetry of the instanton weight $\exp(2\pi i\tau)$ in the Euclidean path integral. The full SL(2, $\mathbb{Z}$) duality implies that the sum of all multi-instanton contributions to any observable quantity must be a modular-invariant function of τ . Evidence for this comes from the protected R^4 corrections to the supergravity action [52].

这清晰表明，在欧几里得路径积分中，$\tau \to \tau + 1$ 是瞬子权重 $\exp(2\pi i\tau)$ 的对称性。完整的 SL(2, $\mathbb{Z}$) 对偶意味着，所有多瞬子对任意可观测物理量的贡献之和，必须是关于 τ 的模不变函数。这一点的证据来自超引力作用量中受保护的 R^4 修正 [52]。

Consider next the D-string whose tension is $T_{\mathrm{D1}} = T_{\mathrm{F}}/g_s$, with $T_{\mathrm{F}} = 1/(2\pi\alpha')$ the fundamental-string tension. The S-duality $\tau \to -1/\tau$ exchanges the NS-NS and RR 2-form fields and inverts the coupling g_s when $C = 0$. We see that the tension formula is consistent with the swapping of F-strings and D-strings, thus validating the S-duality conjecture.

接下来考虑 D 弦，其张力为 $T_{\mathrm{D1}} = T_{\mathrm{F}}/g_s$ ，其中 $T_{\mathrm{F}} = 1/(2\pi\alpha')$ 是基本弦张力。S 对偶 $\tau \to -1/\tau$ 交换了 NS-NS 和 RR 2 形式场，当 $C = 0$ 时对耦合 g_s 做反转。我们可以看到张力公式与 F 弦和 D 弦的交换一致，从而验证了 S 对偶猜想。

More generally, the duality predicts an $\mathrm{SL}(2,\mathbb{Z})$ orbit of (p,q) strings which is isomorphic to that of dyons in $N = 4$ SYM. The string tension is given by the same invariant mass formula Eq. (63) with the replacements $(n_e, n_m) \to (p,q)$ and with $v \to T_F/\sqrt{4\pi g_s}$. Let us see how the D-brane action (105) leads to the correct spectrum of $(p,1)$ strings. Start with a D-string wrapping a circle of unit radius, say in the x^1 direction. Assume for simplicity a constant axion-dilaton $\tau = i/g_s$. The D-string action in static gauge reads

更一般地，该对偶预言了 (p,q) 弦的一个 $\mathrm{SL}(2,\mathbb{Z})$ 轨道，它同构于 $N = 4$ 超杨米尔斯理论中磁单极子-电荷的轨道。弦张力由相同的不变质量公式即式 (63) 给出，只需做替换 $(n_e, n_m) \to (p,q)$ 和 $v \to T_F/\sqrt{4\pi g_s}$ 。我们来看 D 膜作用量 (105) 如何导出 $(p,1)$ 弦的正确谱。从缠绕单位半径圆周的 D 弦开始，例如沿 x^1 方向缠绕。为简化起见，假设轴子-伸缩子场 $\tau = i/g_s$ 是常数。静态规范下的 D 弦作用量为

$$S_{\mathrm{D1}} = -T_{\mathrm{D1}} \int dx^0 dx^1 \sqrt{-(\det \hat{g} + \mathcal{F}^2)} + T_{\mathrm{D1}} \int \widehat{C}_2$$

where $\hat{g}_{\alpha\beta}$ is the induced metric, $\widehat{C}_2$ is the pullback of the RR 2-form, and $\mathcal{F}$ is the (01) component of the 2-form $\widehat{B}_2 + F$. The worldsheet gauge field has only one degree of freedom, the Wilson line $T_{\mathrm{F}} \oint dx^1 A_1 \in [0, 2\pi]$. The momentum conjugate to A_1 is constant and quantized:

其中 $\hat{g}_{\alpha\beta}$ 是诱导度规，$\widehat{C}_2$ 是 RR 2 形式的拉回，$\mathcal{F}$ 是 2 形式 $\widehat{B}_2 + F$ 的 (01) 分量。世界面规范场只有一个自由度，即威尔逊线 $T_{\mathrm{F}} \oint dx^1 A_1 \in [0, 2\pi]$ 。对应 A_1 的共轭动量是常数且满足量子化条件:

$$\pi_{A_1} = \frac{T_{\mathrm{D1}} \mathcal{F}}{\sqrt{-(\det \hat{g} + \mathcal{F}^2)}} = pT_{\mathrm{F}} \Rightarrow \frac{\mathcal{F}}{\sqrt{-\det \hat{g}}} = \frac{pT_{\mathrm{F}}}{\sqrt{T_{\mathrm{D1}}^2 + p^2 T_{\mathrm{F}}^2}} \tag{109}$$

for integer p . Performing the partial Legendre transformation $S'_{\mathrm{D1}} = S_{\mathrm{D1}} - pT_{\mathrm{F}} \int F$ to eliminate A_1 from the action [24] gives after some algebra:

其中 p 为整数。对作用量 [24] 做部分勒让德变换 $S'_{\mathrm{D1}} = S_{\mathrm{D1}} - pT_{\mathrm{F}} \int F$ 以消去 A_1，经过代数运算得到:

$$S'_{\mathrm{D1}} = T_{(p,1)} \int dx^0 dx^1 \sqrt{-\det \hat{g}} + T_{\mathrm{D1}} \int \widehat{C}_2 + pT_{\mathrm{F}} \int \widehat{B}_2, \tag{110}$$

[24] Such a partial Legendre transform is known in classical mechanics as a Routhian. It is useful for eliminating cyclic variables from the action.

[24] 这种部分勒让德变换在经典力学中被称为劳斯变换，它可用于从作用量中消去循环变量。

with $T_{(p,1)} = \left(T_{\mathrm{D1}}^2 + p^2 T_{\mathrm{F}}^2\right)^{1/2}$. This is the action of a $(p, 1)$ string with the tension predicted by $\mathrm{SL}(2, \mathbb{Z})$ duality. Repeating the calculation with $C = \operatorname{Re} \tau \neq 0$ gives the more general formula (63). To obtain arbitrary (p, q) strings, one must start with q D-strings and endow each of them with p/q units of π_{A_1} . We will see why such fractional momenta are allowed in the coming section.

其中 $T_{(p,1)} = \left(T_{\mathrm{D1}}^2 + p^2 T_{\mathrm{F}}^2\right)^{1/2}$ 。这就是张力符合 $\mathrm{SL}(2, \mathbb{Z})$ 对偶预言的 $(p, 1)$ 弦的作用量。对 $C = \operatorname{Re} \tau \neq 0$ 重复该计算可得到更一般的公式 (63)。要得到任意的 (p, q) 弦，我们需要从 q 个 D 弦出发，给每个 D 弦赋予 p/q 单位的 π_{A_1} 。我们会在下一节说明这类分数动量为何是允许的。

The analogy with the BPS spectrum of $N = 4$ SYM is not a sheer coincidence. The SYM dyons are actually realized by (p, q) strings stretching between parallel D3 branes, so that the duality of $N = 4$ SYM follows from the $\mathrm{SL}(2, \mathbb{Z})$ duality of type-IIB string theory (but not the other way around). Note that a D3-brane in the ground state couples only to the self-dual 4-form and is invariant under $\mathrm{SL}(2, \mathbb{Z})$ transformations.

与 $N = 4$ 超对称杨-米尔斯 BPS 谱的类比绝非巧合。杨-米尔斯磁单极子实际上由平行 D3 膜之间伸展的 (p, q) 弦实现，因此 $N = 4$ 超对称杨-米尔斯的对偶性来源于 IIB 型弦论的 $\mathrm{SL}(2, \mathbb{Z})$ 对偶性 (反之则不成立)。注意基态的 D3 膜仅耦合自对偶四形式，且在 $\mathrm{SL}(2, \mathbb{Z})$ 变换下保持不变。

Consider next the D5 brane which is a magnetic source for C_2 . Since $(B_2, -C_2)$ transforms as a doublet of $\mathrm{SL}(2,\mathbb{Z})$, duality also predicts that there exists an orbit of supersymmetric (p,q) five-branes. Set again for simplicity the RR scalar $C = 0$. Dirac's minimal quantization, Eq. (94), and the BPS relation between tension and charge determine the tension of the (1,0) five-brane which is a magnetic source for the NS-NS field B_2 :

接下来考虑作为 C_2 磁源的 D5 膜。由于 $(B_2, -C_2)$ 在 $\mathrm{SL}(2,\mathbb{Z})$ 下变换为二重态，对偶性也预言存在超对称 (p,q) 五膜的轨道。为简化，再次设定 RR 标量 $C = 0$ 。狄拉克最小量子化 (式 (94)) 以及张力与电荷间的 BPS 关系确定了作为 NS-NS 场 B_2 磁源的 (1,0) 五膜的张力:

$$T_{\mathrm{NS5}} = \frac{2\pi^2\alpha'}{\kappa_{10}^2}. \tag{111}$$

As explained below Eq. (94), the κ_{10}^{-2} scaling is characteristic of ordinary solitons, i.e., smooth localized solutions of the classical supergravity equations. The classical solutions of superstring field theory correspond to superconformal σ -models on the string worldsheet. Such a model does exist for the solitonic NS5 brane [53], although our understanding of it remains incomplete. We will revisit this NS5-brane solution in section ”Special Effects.”

正如式 (94) 下方解释的，κ_{10}^{-2} 标度是普通孤子 (即经典超引力方程的光滑局域解) 的特征。超弦场论的经典解对应弦世界面上的超共形 σ 模型。孤子 NS5 膜确实存在这样的模型 [53]，但我们对它的理解仍不完整。我们会在“特殊效应”章节重新讨论这个 NS5 膜解。

Seven-branes must also form $\mathrm{SL}(2,\mathbb{Z})$ orbits. To understand why recall that the D7 brane is a conical point defect in the transverse complex plane with axion monodromy $\tau \to \tau + 1$ that leaves invariant the D-instanton weight $\exp(2\pi i\tau)$. Since the choice of $\mathrm{SL}(2,\mathbb{Z})$ frame is arbitrary, we may trade the fundamental string for a (p,q) string which has the effect of conjugating the monodromy matrix:

七膜也必须构成 $\mathrm{SL}(2,\mathbb{Z})$ 轨道。究其原因，回忆 D7 膜是横截复平面上的锥点缺陷，其轴子单畴变换 $\tau \to \tau + 1$ 保持 D 瞬子权重 $\exp(2\pi i\tau)$ 不变。由于 $\mathrm{SL}(2,\mathbb{Z})$ 参考系的选择是任意的，我们可以将基本弦替换为 (p,q) 弦，这会对单畴矩阵进行共轭变换:

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} pr \\ qs \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} pr \\ qs \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} pr \\ qs \end{pmatrix}^{-1} = \begin{pmatrix} 1-pq & p^2 \\ -q^2 & 1+pq \end{pmatrix}. \tag{112}$$

So $[p,q]$ seven-branes must exist for each pair of relative primes, but only the D7 brane is amenable to a perturbative treatment. It is sometimes convenient to think of $\mathrm{SL}(2,\mathbb{Z})$ as large diffeomorphisms of a 2-torus in a twelve-dimensional theory dubbed ”F-theory” [54]. There is however no local supersymmetric action in twelve dimensions, so this is no more than a mathematical device.

因此，每一对互素整数都对应存在一个 $[p,q]$ 七膜，但只有 D7 膜可以用微扰方法处理。有时将 $\mathrm{SL}(2,\mathbb{Z})$ 看作十二维“F 理论”中二维环面的大微分同胚会很方便 [54]。但十二维中不存在局域超对称作用量，因此这只不过是一种数学工具。

Type IIA and M Theory

IIA 型弦理论与 M 理论

By contrast, a local action does exist in eleven dimensions, the $11d$ supergravity of Cremmer, Julia, and Scherk [24]. As we saw in section "Effective Actions," type-IIA supergravity is obtained from it by dimensional reduction. The Kaluza-Klein relation

相比之下，十一维中确实存在局域作用，即 Cremmer、Julia 和 Scherk 提出的 $11d$ 超引力 [24]。正如我们在“有效作用量”一节中所见，IIA 型超引力正是通过维度约化从它得到的。卡鲁扎-克莱因关系

$$\frac{1}{\kappa_{10}^2} = \frac{2\pi r_{10}}{\kappa_{11}^2} \tag{113}$$

suggests at first sight that the strongly coupled type-IIA sting theory corresponds to the small-radius limit, but as we will see the dimensionless ratio, $r_{10}\kappa_{11}^{-2/9}$ is actually large. So, the limit looks like a theory in eleven dimensions which has been given the provisional name "M theory." Contrary however to the case of type IIB whose strong-coupling limit was another weakly coupled string theory, here quantum M theory cannot be independently defined.

初看表明强耦合 IIA 型弦理论对应小半径极限，但我们会看到，无量纲比 $r_{10}\kappa_{11}^{-2/9}$ 实际上是大的。因此，该极限看起来是一个十一维理论，被暂命名为“M 理论”。不过，与强耦合极限是另一个弱耦合弦理论的 IIB 型弦不同，量子 M 理论无法被独立定义。

Nevertheless, much is known about the half-BPS excitations of this putative theory [48,55]. To begin with, the 3-form of 11-dimensional supergravity can couple consistently to a supersymmetric membrane [56]. Wrapped around the eleventh dimension, this looks like a $10d$ string. Second, there exists a five-brane soliton that is charged under the (Hodge dual) magnetic 6-form [57, 58]. Wrapped around the circle, it looks like a four-brane in $10d$. Finally, one has the standard Kaluza-Klein (KK) modes as well as the Kaluza-Klein monopole whose three-dimensional transverse space is described by the Taub-NUT solution of Einstein's equations [59, 60]. The one-to-one correspondence between these excitations and the branes of type-IIA string theory is shown in Table 3.

尽管如此，关于这个假想理论的半 BPS 激发，我们已经了解很多 [48,55]。首先，十一维超引力的 3-形式可以与超对称膜一致耦合 [56]。缠绕第十一维后，它看起来就是一根 $10d$ 弦。其次，存在一个五膜孤子，它是 (霍奇对偶) 磁 6-形式的荷源 [57, 58]。缠绕圆周后，它看起来就是 $10d$ 中的一个四膜。最后，除了标准卡鲁扎-克莱因 (KK) 模式，还有卡鲁扎-克莱因单极，其三维横向空间由爱因斯坦方程的 Taub-NUT 解描述 [59, 60]。这些激发与 IIA 型弦理论的膜之间存在一一对应关系，见表 3。

Missing from the table is the D8 brane that sources the constant RR field strength F_{10} . This was identified in section "Effective Actions" with the Romans mass of type-IIA supergravity which has no local lift to eleven dimensions. [25]

表中未列出的是产生常数 RR 场强 F_{10} 的 D8 膜。它在“有效作用量”一节中被对应到 IIA 型超引力的 Romans 质量，该质量无法局域提升至十一维。[25]

The tensions in Table 3 are those coupling to the ten-dimensional Einstein-frame metric, with the quadratic supergravity action multiplied by $1/2\kappa_{10}^2$. The type-IIA entries have been computed in section ”D-Branes as Solitons.” To compare with M theory, we need to express r_{10} and κ_{11} in terms of the string-theory parameters α' and κ_{10} . This can be done by using Eq. (113) and by identifying the D-particle mass with the mass of the first KK excitation. Then, the remaining M-theory entries are predictions of the conjectured IIA/M-theory duality.

表 3 中的张力是耦合十维爱因斯坦框架度规的张力，其中二次超引力作用量乘以 $1/2\kappa_{10}^2$ 。IIA 型的各条目已在“D 膜作为孤子”一节中计算完成。为了和 M 理论比较，我们需要用弦理论参数 α' 和 κ_{10} 表示 r_{10} 和 κ_{11} 。这可以通过式 (113) 并将 D 粒子质量等同于第一类 KK 激发的质量来完成。剩余的 M 理论条目就是这个 IIA/M 理论对偶猜想的预言。

Although we have no independent formulation of M-theory, we can still compute the right column of Table 3 from first principles as I now explain. Thanks to BPS saturation, one can think of tension or charge interchangeably. The M2 brane and the M5 brane are electric/magnetic sources of the 3-form in eleven dimensions, so they must obey Dirac quantization:

虽然我们没有 M 理论的独立表述，但我们仍可以像我接下来说明的那样，从第一性原理出发计算表 3 的右列。得益于 BPS 饱和，张力和电荷可以互换看待。M2 膜和 M5 膜分别是十一维中 3-形式的电/磁源，因此它们必须满足狄拉克量子化条件:

$$2\kappa_{11}^2 T_2^{\mathrm{M}} T_5^{\mathrm{M}} = 2\pi n \tag{114}$$

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[25] But can be described by a nonlocal extension of supergravity known as generalized exceptional field theory [61].

[25] 但可以通过超引力的非局部推广即广义例外场论来描述 [61]。

Table 3 Correspondence between half-BPS excitations of type-IIA string theory and of M theory compactified on a circle. As explained in the main text, the two sides can be computed independently, but only one of the five relations is real evidence for the existence of the 11th dimension

表 3 IIA 型弦理论的半 BPS 激发与紧致化在圆周上的 M 理论的半 BPS 激发之间的对应关系。正文中解释过，两边都可以独立计算，但五个关系中只有一个是第十一维存在的切实证据。

Tension	Type-IIA	M theory on S^1	Tension
$\frac{\sqrt{\pi}}{\kappa_{10}}\left(2\pi\sqrt{\alpha'}\right)^3$	D-particle	KK modes	$\frac{1}{r_{10}}$
$T_{\mathrm{F}}=\frac{1}{2\pi\alpha'}$	String	Wrapped membrane	$2\pi r_{10}\left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{1/3}$
$\frac{\sqrt{\pi}}{\kappa_{10}}\left(2\pi\sqrt{\alpha'}\right)$	D2 brane	Membrane	$T_2^M=\left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{1/3}$
$\frac{\sqrt{\pi}}{\kappa_{10}}\left(2\pi\sqrt{\alpha'}\right)^{-1}$	D4 brane	Wrapped five-brane	$r_{10}\left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{2/3}$
$\frac{2\pi^2\alpha'}{\kappa_{10}^2}$	NS5 brane	Five-brane	$T_5^M=\frac{1}{2\pi}\left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{2/3}$
$\frac{\sqrt{\pi}}{\kappa_{10}}\left(2\pi\sqrt{\alpha'}\right)^{-3}$	D6-brane	KK monopole	$\frac{2\pi^2 r_{10}^2}{\kappa_{11}^2}$

String theory predicts that $n=1$. Alternatively, this follows from the ”completeness hypothesis” [62,63] which states that all gauge charges obeying Dirac quantization must be in the spectrum. [26] This is one of the best-motivated ”principles” of quantum gravity, so we will admit it.

弦理论预言 $n=1$ 。另外，这也可以由“完备性假设”[62,63] 推出，该假设指出所有满足狄拉克量子化的规范荷都必须存在于谱中。[26] 这是量子引力中动机最充分的“原理”之一，因此我们认可它。

The condition (114) is not sufficient to determine each tension separately. But a second relation verified by the entries in Table 3:

条件 (114) 不足以单独确定每个张力。但表 3 中的条目满足第二个关系:

$$2\pi T_5^{\mathrm{M}}=\left(T_2^{\mathrm{M}}\right)^2, \tag{115}$$

also follows from topological considerations in eleven dimensions [66]. The main character of the argument is the Chern-Simons term in the action (67), which can be written as a twelve-dimensional integral:

它也可以从十一维的拓扑考虑中推导出来 [66]。该论证的核心是作用量 (67) 中的陈-西蒙斯项，它可以写成十二维积分:

$$-\frac{1}{12\kappa_{11}^2}\int_{\mathcal{M}} F_4\wedge F_4\wedge F_4 \tag{116}$$

with $\mathcal{M}$ is an open manifold whose boundary is eleven-dimensional spacetime. Now, Dirac quantization implies that the minimum flux through any closed 4-cycle is $T_2^{\mathrm{M}}\int F_4=2\pi$. The Euclidean action (116), on the other hand, should not depend on the choice of $\mathcal{M}$, which is the case provided that

带 $\mathcal{M}$ 的是开流形, 其边界为十一维时空。狄拉克量子化表明, 任意闭 4-链上的最小通量为 $T_2^{\mathrm{M}}\int F_4=2\pi$ 。另一方面，欧几里得作用量 (116) 不应依赖 $\mathcal{M}$ 的选取，当满足以下条件时该结论成立

$$\frac{1}{2\kappa_{11}^2}\left(\frac{2\pi}{T_2^{\mathrm{M}}}\right)^3=2\pi m \tag{117}$$

[26] The completeness hypothesis is related to the absence of global symmetries including n -form and non-invertible symmetries; see [64,65].

> [26] 完备性假设与不存在整体对称性 (包括 n -形式对称性和不可逆对称性) 相关；参见 [64,65]。

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for some $m \in \mathbb{Z}$. Choosing $m = 1$ gives precisely Eq. (115), as can be seen by using Dirac quantization to eliminate from this equation T_5^{M} .

> 对于某个 $m \in \mathbb{Z}$ 成立。选取 $m = 1$ 后恰好得到式 (115)，这可以通过狄拉克量子化消去该式中的 T_5^{M} 得到验证。

The choice $m = 1$ is in the same spirit but does not follow immediately from the completeness hypothesis. One can argue that if $m > 1$ Euclidean configurations with fractional flux have a well-defined action and should be included. It would be interesting to relate this to global symmetries.

> $m = 1$ 的选取思路与完备性假设一致，但无法直接从该假设推出。我们可以论证，若 $m > 1$ 欧几里得构型带分数通量，其作用量是良定义的，因此应当被纳入。将其与整体对称性联系起来会是一项有意义的研究。

Let us now take stock of what we have learned. Dirac quantization and the single-valuedness of the Chern-Simons action give four consistency conditions that must hold independently on each side of the type-IIA/M-theory duality. Combined with the completeness hypothesis, which is verified in string theory and is assumed in M-theory, they provide four relations among the six entries of Table 3. Since we used the top entry to establish the $(r_{10}, \kappa_{11}) \leftrightarrow (\alpha', \kappa_{10})$ dictionary, only one relation is left as an independent test of the duality conjecture. We can choose it to be

> 现在我们来梳理已得到的结论。狄拉克量子化与陈-西蒙斯作用量的单值性给出了四个自洽条件，在 IIA 型/M 理论对偶的两端都必须分别满足。结合弦论中已验证、M 理论中假设成立的完备性假设，这四个条件给出了表 3 六个参量之间的四个关系。由于我们已经用首行参量建立了 $(r_{10}, \kappa_{11}) \leftrightarrow (\alpha', \kappa_{10})$ 对应词典，仅剩下一个关系可作为对偶猜想的独立检验。我们可以选取该关系为

$$T_{\mathrm{D0}} T_{\mathrm{F}} = 2\pi T_{\mathrm{D2}}. \tag{118}$$

This is a topological relation in M theory: it follows from the fact that the wrapped M2 brane is the fundamental string and the KK excitation is the D-particle. From the type-IIA perspective, on the other hand, this relation follows from T-duality or from the cylinder calculation of section "D-Brane Tension and Charge," but it has no geometric meaning. This is the evidence from Table 3 for the existence of the eleventh dimension.

> 这是 M 理论中的拓扑关系: 它由以下事实导出: 缠绕的 M2 膜就是基本弦，KK 激发就是 D 粒子。另一方面，从 IIA 型弦论的视角，该关系可由 T 对偶或“D 膜张力与电荷”一节的柱面计算得到，但它没有几何意义。这就是表 3 中支持第十一维存在的证据。

Interactions Between D-Branes

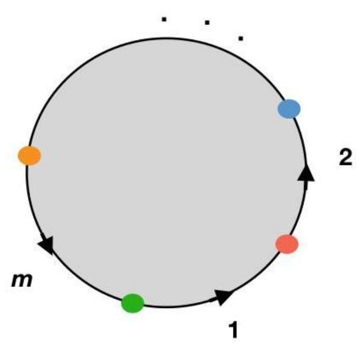
D 膜之间的相互作用

The massless degrees of freedom of an isolated D-brane are a supersymmetric Maxwell multiplet. When many D-branes are present, one must associate to the open strings a matrix-valued wavefunction λ_{ij} such that $|\lambda_{ij}|^2$ is the probability that the string stretches from the i th to the j th D-brane. The indices i, j are called for historical reasons Chan-Paton factors. Classical interactions of open strings are proportional to the trace of the product of their matrix wavefunctions, as illustrated in Fig. 7. General amplitudes have a trace for each boundary of the Riemann surface. When N D-branes are identical and coincident, the amplitudes must be invariant under $U(N)$ rotations of the Chan-Paton factors. The worldvolume gauge theory thus becomes non-abelian SYM with the D-branes coordinates matrix valued. In contrast to ordinary gravitational solitons like black holes which have little or no internal structure, D-brane solitons hide a rich SYM theory in their interior. It is this unique feature that opens a new window into the microscopic structure of spacetime. We will now introduce some features of composite D-brane systems that will be developed in much greater detail elsewhere in this volume.

孤立 D 膜的无质量自由度是一个超对称麦克斯韦多重态。当存在多个 D 膜时，必须为开弦关联一个矩阵值波函数 λ_{ij}，其中 $|\lambda_{ij}|^2$ 是该弦连接第 i 个 D 膜与第 j 个 D 膜的概率。出于历史原因，指标 i, j 被称为陈-帕顿因子。如图 7 所示，开弦的经典相互作用正比于其矩阵波函数乘积的迹。一般振幅会对黎曼曲面的每个边界取一次迹。当 N 个 D 膜全同且重合时，振幅在陈-帕顿因子的 $U(N)$ 转动下保持不变。因此世界体积规范理论成为非阿贝尔超对称杨-米尔斯理论，其中 D 膜坐标为矩阵值。和黑洞这类几乎没有内部结构的普通引力孤子不同，D 膜孤子内部蕴含着丰富的超对称杨-米尔斯理论。正是这一独特特性为我们打开了一扇探索时空微观结构的新窗口。接下来我们将介绍复合 D 膜系统的一些性质，本卷其他部分会对这些性质进行更详尽的阐释。

Fig. 7 Disk diagram for the interaction of m open strings. The colored dots represent D-branes. The amplitude is proportional to $\mathrm{tr}\left(\lambda^1\lambda^2\cdots\lambda^m\right)$

图 7 m 根开弦相互作用的圆盘图。彩色点代表 D 膜。振幅正比于 $\mathrm{tr}\left(\lambda^1\lambda^2\cdots\lambda^m\right)$



Scattering D-Particles

D 粒子散射

Scattering strings at high energy cannot probe distances below the minimal size of a fundamental string $\sim \sqrt{\alpha'}$. Scattering strings off D-branes does not help either, when one smashes a nail with a hammer it is the hammer that limits the resolution. One can however probe substringy distances by scattering slow D-branes off one another, as I now explain.

高能散射弦无法探测小于基本弦最小尺度的距离 $\sim \sqrt{\alpha'}$ 。用弦散射 D 膜也无济于事，这就好比用锤子钉钉子，分辨率的限制来自锤子本身。不过，我接下来会说明，慢运动 D 膜的对散射可以探测弦尺度以下的距离。

We focus on D-particles, the extension to other compact Dp branes is easy. At weak coupling ($g_s \ll 1$), the D-particles are heavy, and their Compton wavelength $\sim g_s\sqrt{\alpha'}$ is much smaller than the fundamental-string scale. But probing it brings in strong-gravity effects since the Schwarzschild radius of the D-particle is of the same order as its Compton wavelength, $r_{\mathrm{S}} \sim \kappa_{10}^2 T_{\mathrm{D0}} \sim g_s\sqrt{\alpha'}$. The best one can do [67] is probe the $11d$ Planck length, which is larger than the Schwarzschild radius but still much below the fundamental-string length $\left(\ell_{11} = \kappa_{11}^{9/2} \sim g_s^{3/4}\sqrt{\alpha'}\right)$.

我们聚焦于 D 粒子，将结论推广到其他紧致 Dp 膜是很容易的。在弱耦合下 ($g_s \ll 1$)，D 粒子质量很大，其康普顿波长 $\sim g_s\sqrt{\alpha'}$ 远小于基本弦尺度。但探测该波长会引入强引力效应，因为 D 粒子的史瓦西半径和它的康普顿波长同阶，即 $r_{\mathrm{S}} \sim \kappa_{10}^2 T_{\mathrm{D0}} \sim g_s\sqrt{\alpha'}$ 。目前能做到的最佳探测 [67] 结果是达到普朗克长度 $11d$ ，该长度比史瓦西半径大，但仍远小于基本弦长度 $\left(\ell_{11} = \kappa_{11}^{9/2} \sim g_s^{3/4}\sqrt{\alpha'}\right)$。

To show this, consider the thought experiment illustrated in Fig. 8. A D-paricle moving on a linear trajectory in the direction x^1 passes near a second D-particle at rest with impact parameter b along the x^2 direction. We are interested in the phase shift, δ , as function of b and the relative velocity $u > 0$. The leading contribution comes from the annulus diagram with (DD) boundary conditions for $X^{2,\cdots,9}$ and mixed boundary conditions for $X^{0,1}$:

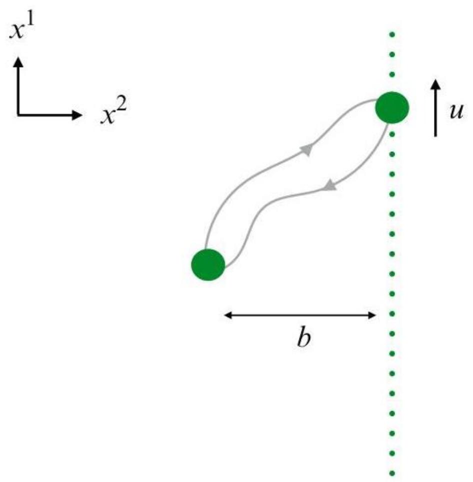
为说明这一点，我们考虑图 8 所示的思想实验: 一个 D 粒子沿 x^1 方向做直线运动，从一个静止的第二个 D 粒子附近经过，沿 x^2 方向的碰撞参数为 b 。我们关注相移 δ ，它是 b 和相对速度 $u > 0$ 的函数。领头贡献来自环形图，其中 $X^{2,\cdots,9}$ 满足 (DD) 边界条件，$X^{0,1}$ 满足混合边界条件:

$$\partial_1 X^0 = X^1 = 0 \ \text{ at } \sigma^1 = 0;$$

$$\partial_1 X'^0 = X'^1 = 0 \ \text{ at } \sigma^1 = \pi. \tag{119}$$

Fig. 8 Scattering of two D-particles (green dots). A pair of virtual open strings (in light gray color) can materialize from the vacuum and slow down the motion

图 8 两个 D 粒子 (绿色点) 的散射。一对虚开弦 (浅灰色) 可从真空中产生，使运动减速



Here, $X'^{\pm} = e^{\mp\pi\varepsilon}X^{\pm}$ are the coordinates in the rest frame of the moving D-paricle expressed in terms of the rapidity:

此处，$X'^{\pm} = e^{\mp\pi\varepsilon}X^{\pm}$ 是运动 D 粒子静止系中用快度表示的坐标:

$$\pi\varepsilon = \operatorname{arctanh}(u)\,. \tag{120}$$

The reader will notice the similarity with the analysis of D-strings making an angle; see section ”Classical Motion of Open and Closed Strings.” The two problems are related by rotating to imaginary time and identifying $\pi\varepsilon$ with an imaginary angle ϑ . The mode expansion of the coordinates $\left(X^{\pm},\psi^{\pm}\right)$ and their contribution to the annulus diagram can be worked out easily and are left as an exercise. The final result for the phase shift is [17]

读者会发现这和成一定角度的 D 弦分析相似，参见“开弦与闭弦的经典运动”一节。这两个问题可通过转到虚时间、并将 $\pi\varepsilon$ 等同于虚角 ϑ 建立关联。坐标 $\left(X^{\pm},\psi^{\pm}\right)$ 的模展开及其对环形图的贡献很容易推导，留作练习。相移的最终结果为 [17]

$$\delta(b,\varepsilon) = -\int_0^{\infty}\frac{dt}{t}e^{-b^2t/4\pi\alpha'}Z_{\text{open}}\left(\varepsilon, q = e^{-\pi t}\right),$$

$$\text{where } Z_{\text{open}} = -\frac{1}{2}\sum_{j=2,3,4}(-)^j\frac{\theta_j\left(\frac{1}{2}\varepsilon t \mid q\right)}{\theta_1\left(\frac{1}{2}\varepsilon t \mid q\right)}\frac{\theta_j^3(0\mid q)}{\eta^9(q)} \tag{121}$$

and θ_j are the Jacobi theta functions. This generalizes Polchinski’s calculation of the static force, Eq. (92), to moving D0 branes.

其中 θ_j 是雅可比 θ 函数。这将 Polchinski 对静电力的计算 (式 (92)) 推广到了运动 D0 膜。

Using standard Jacobi identities, one can write Z_{open} in the equivalent form

利用标准雅可比恒等式，可以将 Z_{open} 改写为等价形式

$$Z_{\text{open}}(\varepsilon, t) = \frac{\theta_1^4\left(\frac{1}{4}\varepsilon t \mid q\right)}{\theta_1\left(\frac{1}{2}\varepsilon t \mid q\right)\eta^9(q)} \tag{122}$$

from which the small-velocity limit is easier to extract. The product formula

从小速度极限中提取结果会更方便。乘积公式

$$\frac{\theta_1(z \mid q)}{\eta(q)} = 2\sin(\pi z)\, q^{\frac{1}{12}} \prod_{n=1}^{\infty}\left(1 - 2q^n \cos(2\pi z) + q^{2n}\right)$$

leads indeed after a little algebra to the expansion

经过简单代数运算后确实可得到展开式

$$Z_{\text{open}} = \frac{(\pi\varepsilon t)^3}{16} + O\left(\varepsilon^7\right) \quad \text{where} \quad (\pi\varepsilon)^3 = u^3 + u^5 + O\left(u^7\right). \tag{123}$$

Inserting in Eq. (121) and performing the integral gives

代入式 (121) 并完成积分可得

$$\delta(b, \varepsilon) = \left(\frac{2\pi\alpha'}{b^2}\right)^3 (\pi\varepsilon)^3 + O\left(\varepsilon^7\right). \tag{124}$$

In order to interpret the result, we define the interaction energy $E_{\text{int}}(u, r)$, where r is the instantaneous distance between the D-particles, via an integral transform:

为了解释该结果，我们定义相互作用能 $E_{\text{int}}(u, r)$，其中 r 是 D 粒子之间的瞬时距离，通过积分变换得到:

$$\delta(b, u) = \int_{-\infty}^{\infty} d\tau E_{\text{int}}\left(\sqrt{u^2\tau^2 + b^2}, u\right) = \frac{2}{u}\int_b^{\infty} dr \frac{r}{\sqrt{r^2 - b^2}} E_{\text{int}}(r, u).$$

(125)

The phase shift (124) gives

相移 (124) 给出

$$E_{\text{int}}(r, u) = \frac{15}{16}\frac{(2\pi\alpha')^3}{r^7}\left(u^4 + u^6\right) + O\left(u^8\right). \tag{126}$$

One learns from this expansion (i) that not only the static but also the $O\left(u^2\right)$ force between D-particles vanishes and (ii) that the leading $O\left(u^4\right)$ interaction does not depend on the string scale α'. Both results follow from spacetime supersymmetry or more precisely from the fact that only half-BPS states of the open string contribute at this order to the supertrace [68]. Since only the ground states of the open superstring are BPS, we understand why α' dropped out.

我们从该展开中得到:(i) D 粒子之间不仅静电力为零，$O\left(u^2\right)$ 力也为零；(ii) 领头阶 $O\left(u^4\right)$ 相互作用不依赖于弦标度 α' 。这两个结果都源于时空超对称性，更准确地说，是因为在这个阶上，只有开弦的半 BPS 态对超迹有贡献 [68]。由于只有开超弦的基态是 BPS 态，我们也就明白为什么 α' 会消去了。

We will return to these remarks in a minute, but first let us point out that $\delta(b,\varepsilon)$ also has an imaginary, i.e., absorptive part. This arises from the zeroes of $\theta_1(z\mid q)$ at $z=k\in\mathbb{Z}$ which produce poles in Z_{open} at $t=2k/\varepsilon$ for k odd; see Eq. (122). The integration contour must leave all the poles on the same side in order not to obstruct the rotation to the imaginary- t half-axis which corresponds to imaginary rapidity. The absorptive part is the sum of the residues at these poles, explicitly

我们稍后再继续讨论这些内容，首先需要指出 $\delta(b,\varepsilon)$ 也存在一个虚部，也就是吸收部分。它来源于 $\theta_1(z\mid q)$ 在 $z=k\in\mathbb{Z}$ 处的零点，当 k 为奇数时，这些零点会在 $t=2k/\varepsilon$ 处的 Z_{open} 中产生极点；参见式 (122)。积分围道必须将所有极点留在同一侧，才能顺利旋转到对应虚快度的虚 t 半轴。吸收部分就是这些极点处留数的和，具体表达式为

$$\operatorname{Im}\delta=\frac{1}{2}\sum_{S}\sum_{k\text{ odd}}\frac{1}{k}\exp\left[-\frac{k}{\varepsilon}\left(\frac{b^2}{2\pi\alpha'}+2\pi\alpha' M_S^2\right)\right], \tag{127}$$

where the sum runs over all open-string states S with mass M_S and the even powers of k cancel between bosons and fermions.

其中求和遍及所有质量为 M_S 的开弦态 S ，且 k 的偶次幂在玻色子与费米子之间相互抵消。

This absorptive part expresses the probability $P=e^{-2\operatorname{Im}\delta}$ that a pair of open strings materializes from the vacuum and slows down the relative motion; see Fig. 8.

该吸收部分表示一对开弦从真空中产生并降低相对运动速度的概率 $P=e^{-2\operatorname{Im}\delta}$ ；参见图 8。

The phenomenon is T-dual to Schwinger's pair creation in an electric field, generalized to the case where the charge is carried by open strings [69]. T-duality maps the speed of light to the limiting Born-Infeld electric field. As $u\to 1 \Leftrightarrow \varepsilon\to\infty$, the process is unsuppressed even if the impact parameter is large, $b\gg\sqrt{\alpha'}$. In this ultrarelativistic limit $2\pi\varepsilon\simeq-\log(1-u)$, so the critical impact parameter below which the scattering becomes inelastic reads

这一现象是电场中施温格对产生的 T 对偶，推广到了开弦携带电荷的情况 [69]。T 对偶将光速映射为玻恩-因费尔德极限电场。正如 $u\to 1\Leftrightarrow\varepsilon\to\infty$ 所示，即使碰撞参数很大 $b\gg\sqrt{\alpha'}$ ，该过程也不会被压制。在这个极端相对论极限下 $2\pi\varepsilon\simeq-\log(1-u)$ ，因此散射转变为非弹性散射对应的临界碰撞参数为

$$b_{\text{cr}}\simeq\sqrt{-\alpha'\log(1-u)}\simeq\sqrt{2\alpha'\log\left(s/T_{\text{D0}}^2\right)} \tag{128}$$

where s is the Mandelstam variable of the collision. This $\sim\sqrt{2\alpha'\log(s)}$ growth of the critical impact parameter with s is universal; it was also found in string-string and string-brane collisions at sufficiently high energies [70]. It is a stringy effect, to be distinguished from the gravitational tidal excitations; see e.g., Ref. [71].

其中 s 是碰撞的曼德尔斯坦变量。临界碰撞参数随 s 的这种 $\sim \sqrt{2\alpha' \log(s)}$ 增长是普适的；在足够高能量下的弦-弦碰撞和弦-膜碰撞中也发现了这一结果 [70]。这是弦论特有的效应，区别于引力潮汐激发；例如参见文献 [71]。

Let us go back now to substringy b and low velocity $u \simeq \pi\varepsilon \ll 1$. To suppress the inelastic process, we need $b \gg \sqrt{2\alpha' u}$. The quantum uncertainty of the velocity on the other hand, $\delta u \simeq (T_{\mathrm{D0}} b)^{-1}$, must be much smaller than u so we need $T_{\mathrm{D0}} b u \gg 1$. The smallest b that is consistent with both bounds is given by

现在我们回到亚弦尺度 b 和低速 $u \simeq \pi\varepsilon \ll 1$ 的情况。为了压制非弹性过程，我们需要 $b \gg \sqrt{2\alpha' u}$ 。另一方面，速度的量子不确定度 $\delta u \simeq (T_{\mathrm{D0}} b)^{-1}$ 必须远小于 u ，因此我们需要 $T_{\mathrm{D0}} b u \gg 1$ 。同时满足这两个边界条件的最小 b 由下式给出

$$b_{\min}^3 \sim \frac{\alpha'}{T_{\mathrm{D0}}} \sim \frac{1}{T_2^M} \sim l_{11}^3, \tag{129}$$

where we have used Eq. (118) and the fact that in $11d$ Planck units the membrane tension is an $O(1)$ number; see Table 3. This shows that D-particle scattering can probe the $11d$ Planck scale as announced.

其中我们用到了式 (118)，以及在 $11d$ 普朗克单位下膜张是一个 $O(1)$ 量级的数这一事实；参见表 3。这表明正如之前所说，D 粒子散射可以探测普朗克尺度 $11d$ 。

At energies comparable to those of a stretched open string, $E \sim T_F \ell_{11} \sim g_s^{1/3}$ in string units, string excitations, and higher-order terms of the DBI action can be safely ignored, and the system is described by an effective quantum-mechanical model which is the reduction of SYM from $d = 10$ dimensions to only time. The Hamiltonian of this Matrix Quantum Mechanics reads

当能量与拉伸开弦的能量相当时，在弦单位下为 $E \sim T_F \ell_{11} \sim g_s^{1/3}$ ，弦激发以及 DBI 作用量的高阶项都可以安全忽略，该系统由一个有效量子力学模型描述，这个模型是 SYM 从 $d = 10$ 维约化到仅含时间维度的结果。该矩阵量子力学的哈密顿量为

$$H_{\mathrm{MQM}} = r_{10} \operatorname{Tr}\left(8\pi^2 \ell_{11}^6 \sum_i \Pi^i \Pi^i - \sum_{i,j} \left[Y^i, Y^j\right]^2 - \sum_i \bar{\lambda} \Gamma^i \left[Y^i, \lambda\right]\right) \tag{130}$$

where Π^i are the momenta conjugate to the (appropriately rescaled) coordinates Y^i , the gauge group for n D-particles is $U(n)$, and its role is to project non-singlet states out of the spectrum. This Hamiltonian controls the D0 brane interactions at substringy scales and reproduces, as we saw, their leading $O\left(u^4\right)$ long-range force. Other remarkable properties of H_{MQM} are that it provides a discretization of the supersymmetric membrane [72] and that its spectrum (most likely) includes the threshold bound states that are dual to the higher KK modes of eleven-dimensional supergravity [73, 74].

其中 Π^i 是经适当重整后的坐标 Y^i 对应的共轭动量，n 个 D 粒子的规范群为 $U(n)$ ，其作用是将非单态从能谱中投影去除。该哈密顿量控制子弦论尺度下 D0 膜的相互作用，正如我们所见，它可以重现其领头阶 $O\left(u^4\right)$ 长程力。H_{MQM} 的另一个显著性质是，它给出了超膜的离散化表述 [72]，且其能谱 (极有可能) 包含对应十一维超引力更高 KK 模式的阈值束缚态 [73, 74]。

Ordinary solitons have size comparable to the Compton wavelength of the fundamental quanta. For example, the size of the 't Hooft-Polyakov monopole of section " $N = 4$ Super Yang-Mills" is $\sim 1/gv$, the Compton wavelength of the charged gauge bosons. Since weakly coupled D-branes are smaller than quantum strings, one may suspect that the latter are not the fundamental quanta of gravity. A bold proposal by Banks et al [75] was that Matrix Quantum Mechanics in the $n \to \infty$ limit may serve as the non-perturbative definition of M theory. There is, however, no evidence that at higher energies the Hamiltonian (130) reproduces the rich D-particle dynamics of string theory. Soon after this BFFS proposal a similar but sharper conjecture, the duality between $d = 4$ SYM and quantum gravity in a Anti-de Sitter box, alias AdS/CFT correspondence [76-78], revolutionized the subject.

普通孤子的尺寸与基本量子的康普顿波长相当。例如，章节“$N = 4$ 超杨-米尔斯”中't 霍夫特-波利亚科夫磁单极的尺寸为 $\sim 1/gv$, 恰好是带电规范玻色子的康普顿波长。由于弱耦合 D 膜比量子弦更小，我们有理由猜想量子弦并非引力的基本量子。班克斯等人提出了一个大胆的猜想 [75]: $n \to \infty$ 极限下的矩阵量子力学可以作为 M 理论的非微扰定义。但目前没有证据表明哈密顿量 (130) 能在更高能区重现弦论中丰富的 D 粒子动力学。在这个 BFFS 猜想提出后不久，一个相似但更明确的猜想——即 $d = 4$ 超杨-米尔斯与反德西特背景下量子引力的对偶，也就是 AdS/CFT 对应 [76-78]——彻底革新了这个领域。

Bound States and Moduli Spaces

束缚态与模空间

In section "Dualities," we have seen that the worldvolume gauge field $F = dA$ can endow a D-brane with extra charges. For example, the Wess-Zumino term of the action (105) shows that a Dp brane with a magnetic field couples both to C_{p+1} and to C_{p-1} . And as explained in section "The Power of T-Duality," a D-string with a worldvolume electric field is a $(m, 1)$ string, i.e., it carries m units of F-string charge. We will now discuss new effects that arise because of the non-abelian nature of the worldvolume fields and more generally in composite D-brane systems. Many of these effects are easy to understand in one duality frame but look highly nontrivial in others.

在“对偶性”一节中我们已经看到，世界体积规范场 $F = dA$ 可以赋予 D 膜额外的电荷。例如，作用量 (105) 的韦斯-祖米诺项表明，带有磁场的 Dp 膜同时与 C_{p+1} 和 C_{p-1} 耦合。正如“T 对偶的力量”一节中解释的那样，带有世界体积电场的 D 弦就是 $(m, 1)$ 弦，也就是说，它携带 m 单位的 F 弦电荷。我们现在将讨论由世界体积场的非阿贝尔性质引发的新效应，更一般地说，是复合 D 膜系统中的新效应。这些效应中有许多在某个对偶框架下很容易理解，在其他框架下却显得非常不平凡。

We start with the description of two effects, long strings and D1/D5 bound states, entering the construction of the supersymmetric three-charge black hole which lead to the first microscopic derivation of the Bekenstein-Hawking entropy [79].

我们先介绍两个效应: 长弦和 D1/D5 束缚态，它们进入超对称三电荷黑洞的构造，而该构造第一次从微观出发推导出了贝肯斯坦-霍金熵 [79]。

Long strings Consider n identical D2 branes wrapped around a torus in the (12) plane. Take for simplicity the torus to be orthogonal with radii r_1, r_2 . We work in units $\alpha' = 1$. On the Coulomb branch, the gauge group is $U(1)^n$, and we may switch on magnetic fields on each D2 brane separately. To minimize energy, these fields have to be constant. A convenient gauge is $\left(\mathbf{A}^1, \mathbf{A}^2\right) = \left(\mathbf{a}, \mathbf{b} + \mathbf{B}x^1\right)$, where bold-face symbols denote n -dimensional vectors with one component for each $U(1)$ factor of the gauge group. The Wilson lines a_j and b_j (for $j = 1, \cdots, n$) are periodic with periods $2\pi/r_1$ and $2\pi/r_2$, and each magnetic field B_j must be a multiple of $1/(r_1 r_2)$ in accordance with Dirac quantization. [27] The total energy and D0-brane charge of this composite system can be readily computed from the action (105) with the result:

长弦考虑 n 个相同的 D2 膜缠绕在 (12) 平面的环面上。为简化起见，设这个环面是正交的，半径为 r_1, r_2 。我们采用单位制 $\alpha' = 1$ 。在库仑分支上，规范群为 $U(1)^n$ ，我们可以对每个 D2 膜分别开启磁场。为了最小化能量，这些场必须是常数。一个方便的规范是 $\left(\mathbf{A}^1, \mathbf{A}^2\right) = \left(\mathbf{a}, \mathbf{b} + \mathbf{B}x^1\right)$ ，其中粗体符号表示 n 维向量，每个分量对应规范群的一个 $U(1)$ 因子。威尔逊线 a_j 和 b_j (对应 $j = 1, \cdots, n$) 是周期的，周期为 $2\pi/r_1$ 和 $2\pi/r_2$ ，根据狄拉克量子化条件，每个磁场 B_j 必须是 $1/(r_1 r_2)$ 的整数倍。[27] 这个复合系统的总能量和 D0 膜电荷可以很容易地从作用量 (105) 计算得到，结果为:

$$T_{\mathrm{D2}}\left(4\pi^2 r_1 r_2\right)\sum_{j=1}^{n}\sqrt{1+\frac{m_j^2}{\left(r_1 r_2\right)^2}} \text{ and } \rho_{\mathrm{D0}}\sum_{j=1}^{n} m_j \equiv \rho_{\mathrm{D0}} m \tag{131}$$

where $m_j = r_1 r_2 B_j \in \mathbb{Z}$. The energy of n D2 branes and m D-particles, all widely separated in the transverse space, is $nT_{\mathrm{D2}}\left(4\pi^2 r_1 r_2\right) + mT_{\mathrm{D0}}$. Comparing with Eq. (131) and recalling that $T_{\mathrm{D0}} = (2\pi)^2 T_{\mathrm{D2}}$ shows that our configuration describes n separate D2/D0 bound states.

其中 $m_j = r_1 r_2 B_j \in \mathbb{Z}$。$n$ 个 D2 膜和 m 个 D 粒子都在横空间中广泛分离, 其总能量为 $nT_{\mathrm{D2}}\left(4\pi^2 r_1 r_2\right) + mT_{\mathrm{D0}}$ 。与式 (131) 比较，再结合 $T_{\mathrm{D0}} = (2\pi)^2 T_{\mathrm{D2}}$ 可知，我们的构型描述了 n 个独立的 D2/D0 束缚态。

---

[27] We follow the convention of section ”The Power of T-Duality” so that T-duality trades the gauge field for a D-brane coordinate with no multiplicative factor. In this convention, string endpoints have charge $q = 1/2\pi\alpha'$.

[27] 我们遵循“T 对偶的力量”一节的约定，即 T 对偶将规范场替换为 D 膜坐标，不带有乘性因子。在此约定下，弦端点带电荷 $q = 1/2\pi\alpha'$ 。

For general $\mathbf{B}$, this is not the configuration of lowest energy. This becomes clear after T-dualizing the coordinate x^2 which has the effect of converting the magnetized D2-branes to D-strings with diagonal embeddings $\mathbf{Y}^2 = \mathbf{b} + \mathbf{B}x^1$ and inverting also the radius ($\tilde{r}_2 = 1/r_2$) . For given D2 and D0 charges, n and m , the T-dual configuration of minimal energy is one long D-string winding (n, m) times around the two circles, as illustrated in Fig.9. This is the unique ground state whenever (m, n) are relative primes. More generally, the n D-strings recombine into several longer ones, unless m is a multiple of n in which case they are all free to separate.

对于一般的 $\mathbf{B}$ ，这并不是最低能量的构型。对坐标 x^2 做 T 对偶后就会清楚这一点:T 对偶会将带磁化的 D2 膜转化为带对角嵌入的 D 弦 $\mathbf{Y}^2 = \mathbf{b} + \mathbf{B}x^1$ ， 同时也会反转半径 $(\tilde{r}_2 = 1/r_2)$ 。对于给定的 D2 电荷与 D0 电荷 n 和 m ， 最小能量的 T 对偶构型是一条绕两个圆缠绕 (n, m) 圈的长 D 弦，如图 9 所示。当 (m, n) 互质时，这是唯一的基态。更一般地说，n 条 D 弦会重组为数条更长的 D 弦，除非 m 是 n 的倍数，此时所有 D 弦都可以自由分开。

In the original D2-D0 duality frame, the long string corresponds to the following $U(n)$ gauge field:

在原有的 D2-D0 对偶框架下，长弦对应如下 $U(n)$ 规范场:

$$\mathbf{A}^1 = a\mathbf{1}_{n\times n},\ \mathbf{A}^2 = \left(\frac{2\pi}{nr_2}\right)\mathrm{diag}\,(0, 1, \cdots, n-1) + \left(b + \frac{mx^1}{nr_1r_2}\right)\mathbf{1}_{n\times n}, \tag{132}$$

where $\mathbf{1}_{n\times n}$ is the identity $n \times n$ matrix. For fractional m/n , this is not an admissible $U(1)^n$ gauge bundle. But it becomes admissible if the gauge group is $U(n)$, since

其中 $\mathbf{1}_{n\times n}$ 是单位 $n \times n$ 矩阵。对于分数 m/n ， 这不是一个可容许的 $U(1)^n$ 规范丛。但如果规范群是 $U(n)$ ， 它就变成可容许的，因为

$$q\mathbf{A}_\mu\left(x^1 + 2\pi r_1\right) = \mathcal{U}^{-1}\left[q\mathbf{A}_\mu\left(x^1\right) - i\partial_\mu\right]\mathcal{U} \tag{133}$$

with

其中

$$\mathcal{U}(x) = \mathrm{diag}\left(e^{i(1-n)x^2/nr_2}, e^{ix^2/nr_2}, \cdots, e^{ix^2/nr_2}\right)\mathbf{P}_{1\to 2\cdots\to n} \tag{134}$$

and $\mathbf{P}_{1\to 2\cdots\to n}$ the cyclic permutation matrix. We let the reader verify that $\mathcal{U}(x)$ is indeed a single-valued unitary matrix on the torus. It is actually single-valued also in $SU(n)/\mathbb{Z}_n$, but fractional m/n obstructs its lifting to a good $SU(n)$ bundle. This twisting of the boundary conditions of the gauge field is familiar in the study of gauge theories where it is known as a 't Hooft flux [80].

且 $\mathbf{P}_{1\to 2\cdots\to n}$ 是循环置换矩阵。请读者自行验证 $\mathcal{U}(x)$ 在环面上确实是一个单值幺正矩阵。它实际上在 $SU(n)/\mathbb{Z}_n$ 中也是单值的，但分数 m/n 会阻碍它提升为一个合格的 $SU(n)$ 丛。规范场边界条件的这种扭转在规范理论研究中是常见的，被称为 ‘t 霍夫特通量 [80]。

A similar story holds for the (p, q) strings of section "The Power of T-Duality." After T-dualizing the circle wrapped by the string, this latter becomes a collection of q D-particles sharing p units of momentum in the dual dimension. When p and q are relative primes, the ground state is unique and corresponds to an eleven-dimensional KK graviton with incommensurate momenta in the compact dimensions.

在《T 对偶的力量》一节中的 (p, q) 弦也有类似的情况。对弦缠绕的圆做 T 对偶后，原弦就变为一组 q D 粒子，它们在对偶维度共享 p 单位的动量。当 p 与 q 互质时，基态是唯一的，对应一个十一维 KK 引力子，它在紧致维中具有不可公度动量。

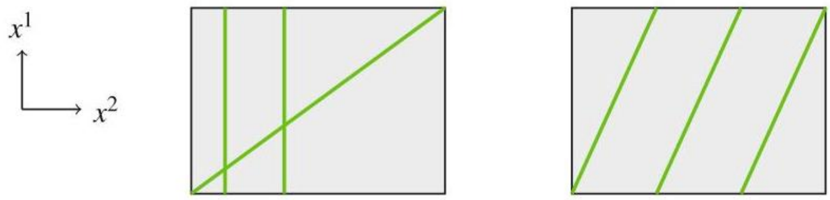


Fig. 9 Three D-strings, one of which winds around the x^2 circle (left), can lower their energy by recombining into one long D-string (right)

图 9 三条 D 弦，其中一条缠绕 x^2 圆 (左图)，可以通过重组为一条长 D 弦降低能量 (右图)

One should note that the non-abelian worldvolume theories are strongly coupled in these low-dimensional systems. Our classical arguments are nevertheless reliable for the above (half-BPS) ground states, thanks to supersymmetry.

需要注意的是，非阿贝尔世界体理论在这些低维系统中是强耦合的。但由于超对称，我们的经典论证对上述 (半 BPS) 基态仍然是可靠的。

1/4-BPS systems Such systems offer less supersymmetry protection but also richer possibilities. They contain open-string sectors with four Neumann-Dirichlet boundary conditions, as explained in section "D-Branes and Spacetime Supersymmetry." A prototype consists of m parallel D9 branes and n D5 branes, but our discussion will be equally valid for systems obtained from D5/D9 by duality transformations. Some systems that enter in important applications are listed in Table 4.

1/4-BPS 系统这类系统提供的超对称保护更少，但也拥有更丰富的可能性。它们包含带有四个诺依曼-狄利克雷边界条件的开弦扇区，正如《D 膜与时空超对称》一节中解释的那样。一个原型系统包含 m 张平行 D9 膜和 n 张 D5 膜，但我们的讨论对 D5/D9 经过对偶变换得到的系统同样成立。一些有重要应用的系统列在表 4 中。

The low-energy theory of a composite $\mathrm{D}p/\mathrm{D}(p+4)$ system is the dimensional reduction of the 6d N = 1SYM that lives on the worldvolume of the parent D5/D9 . The bosonic field content is as follows:

复合 $\mathrm{D}p/\mathrm{D}(p+4)$ 系统的低能理论是母 D5/D9 世界体上存在的 6d N = 1SYM 的维度约化。玻色场内容如下:

- The strings on the Dp branes give the usual $\left(A^{\mu=0,\cdots,p}, Y^{p+1,\cdots,9}\right)$ in the adjoint of the gauge group $U(n)$. The coordinates along the $\mathrm{D}(p+4)$ dimensions transform as a real vector of $SO(4)$ or as a complex doublet of $SU(2)_R \subset SO(4)$. They are part of a hypermultiplet of half-maximal supersymmetry with $SU(2)_R$ the 6d R-symmetry. We define $Z = Y^6 + iY^7$ and $\widetilde{Z}^\dagger = Y^8 + iY^9$ (the components of the doublet) and reserve the symbol Y^j for the coordinates in the remaining $5-p$ directions transverse to all D-branes.

- Dp 膜上的弦给出了规范群 $U(n)$ 伴随表示下的常见 $(A^{\mu=0,\cdots,p}, Y^{p+1,\cdots,9})$。沿 D$(p+4)$ 维的坐标变换为 $SO(4)$ 的实矢量，或 $SU(2)_R \subset SO(4)$ 的复双态。它们属于半最大超对称下超多重态的一部分，R 对称性为 $6d$ $SU(2)_R$。我们定义 $Z = Y^6 + iY^7$ 和 $\widetilde{Z}^\dagger = Y^8 + iY^9$ (双态的分量)，并保留符号 Y^j 表示所有 D 膜剩余垂直方向 $5-p$ 上的坐标。

- The fields on the D$(p+4)$ branes are similar except that they are in the adjoint of $U(m)$. The four real coordinates packaged previously in $Z, \widetilde{Z}$ are now components of the gauge field in the extra dimensions.

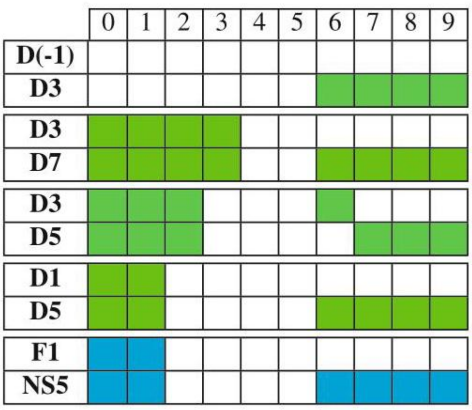
- D$(p+4)$ 膜上的场是类似的，只不过它们属于 $U(m)$ 的伴随表示。此前打包在 $Z, \widetilde{Z}$ 中的四个实坐标，现在是额外维中规范场的分量。

- Finally, we have the open strings stretching between a Dp and a D$(p+4)$ brane. Their (ND) coordinates have integer modes in the Neveu-Schwarz sector (and half-integer in the Ramond sector). It follows from the analysis in sections "The Free Bosonic String" and "Superstrings" that $|0\rangle_{\mathrm{NS}}$ is a $SO(1,p)$ scalar transforming as a chiral spinor of the internal $SO(4)$, i.e., as a doublet of $SU(2)_R$.

- 最后，我们得到了伸展在 Dp 膜和 D$(p+4)$ 膜之间的开弦。它们的 (ND) 坐标在纳维-施瓦茨扇区为整数模，在拉蒙德扇区为半整数模。根据“自由玻色弦”和“超弦”章节的分析可得，$|0\rangle_{\mathrm{NS}}$ 是一个变换为内部 $SO(4)$ 手征旋量 (即 $SU(2)_R$ 双态) 的 $SO(1,p)$ 标量。

Table 4 Brane pairs that are dual to the D5/D9 system. Colored boxes show the dimensions along which each brane extends. The first four configurations are T-dual to D5/D9, whereas D1/D5 and F1/NS5 are related by S-duality. To underline their common $6d$ origin, we chose the non-common dimensions to be always the last four

表 4 对偶于 D5/D9 系统的膜对。有色框标出了每个膜延展的维度。前四种构型是 D5/D9 的 T 对偶，而 D1/D5 和 F1/NS5 通过 S 对偶关联。为强调它们共同的 $6d$ 起源，我们将非公共维度统一放在最后四个位置



We call H and $\widetilde{H}^\dagger$ the complex components of the doublet; they are in the $(n, \bar{m})$ representation of the gauge groups.

我们将 H 和 $\widetilde{H}^\dagger$ 定义为该双态的复分量；它们属于规范群的 $(n, \bar{m})$ 表示。

The classical low-energy action for all these fields is fixed by supersymmetry and gauge invariance. To simplify the discussion, we place the $\mathrm{D}(p+4)$ branes at the origin in transverse space and freeze all fields on their worldvolume. [28] The action includes the standard kinetic terms and a scalar potential given by

所有这些场的经典低能作用量由超对称和规范不变性确定。为简化讨论，我们将 $\mathrm{D}(p+4)$ 膜放在垂直空间原点，固定其世界体积上的所有场。[28] 作用量包含标准动能项和如下给出的标量势:

$$\frac{2}{g^2}V = \sum_{i>j} \left|[Y^i, Y^j]\right|^2 + \sum_i \left(\left|[Y^i, Z]\right|^2 + \left|[Y^i, \widetilde{Z}^\dagger]\right|^2 + \left|Y^i H\right|^2 + \left|Y^i \widetilde{H}^\dagger\right|^2 \right)$$

$$+ \operatorname{tr}\left(HH^\dagger - \widetilde{H}^\dagger \widetilde{H} + [Z, Z^\dagger] - [\widetilde{Z}^\dagger, \widetilde{Z}]\right)\right|^2 + \operatorname{tr} \mid H\widetilde{H} + [Z, \widetilde{Z}]\right|^2.$$

(135)

We have pulled out a factor of g^2 to make it clear that all terms originate from gauge interactions. In the first line, $|M|^2$ stands for $\operatorname{tr}\left(M^\dagger M\right)$, and all entries come from gauge-invariant kinetic terms in six dimensions. The lower line is the contribution of an $SU(2)_R$ triplet of D-terms. Here, H and $\widetilde{H}^\dagger$ are $n \times m$ matrices, so that all matrices inside the traces are $n \times n$.

- 我们提出一个 g^2 因子，以明确所有项都起源于规范相互作用。第一行中，$|M|^2$ 代表 $\operatorname{tr}\left(M^\dagger M\right)$ ，所有项都来自六维下规范不变的动能项。下一行是 D 项的 $SU(2)_R$ 三重态贡献。此处，H 和 $\widetilde{H}^\dagger$ 是 $n \times m$ 矩阵，因此迹内所有矩阵都是 $n \times n$ 。

Since V is a sum of positive contributions, they must all vanish in vacuum. One option is to have all hypermultiplets vanish and allow commuting $\langle Y^i \rangle$ of the scalars in the vector multiplet. Another is to set all scalars in vector multiplets to zero and solve the D-term equations for the hypermultiplets. These two branches of vacua are called, respectively, the Coulomb and the Higgs branch. The hypermultiplets contain $4\left(nm + n^2\right)$ real scalar fields. Since there are $3n^2$ D-term conditions and there is a redundancy of n^2 gauge transformations, the dimension of the Higgs branch is $4mn$. On the Higgs branch, the transverse coordinates Y^i are massive and the Dp branes are bound to the $\mathrm{D}(p+4)$ branes.

由于 V 是正贡献的和，它们在真空中必须全部为零。一种选择是令所有超多重子都为零，允许向量多重子中的标量满足对易的 $\langle Y^i \rangle$ 。另一种选择是将向量多重子中的所有标量设为零，求解超多重子的 D 项方程。这两个真空分支分别称为库仑分支和希格斯分支。超多重子包含 $4\left(nm + n^2\right)$ 个实标量场。由于存在 $3n^2$ 个 D 项条件，且 n^2 个规范变换存在冗余，因此希格斯分支的维数为 $4mn$ 。在希格斯分支上，横向坐标 Y^i 是有质量的，且 Dp 膜束缚于 $\mathrm{D}(p+4)$ 膜。

From the perspective of the $\mathrm{D}(p+4)$ branes, we can understand this bound state as follows. The $U(m)$ non-abelian generalization of the action (105) contains the Wess-Zumino term:

从 $\mathrm{D}(p+4)$ 膜的视角，我们可以按如下方式理解这个束缚态。作用量 (105) 的 $U(m)$ 非阿贝尔推广包含维斯-祖米诺项:

$$\rho_{\mathrm{D}(p+4)} \int \left[C_{p+5} + \frac{1}{2} \operatorname{tr}(F \wedge F)\, C_{p+1} \right]. \tag{136}$$

The integral of $\frac{1}{2}\operatorname{tr}(F \wedge F)$ over the four extra directions of the $\mathrm{D}(p+4)$ brane is a topological invariant equal to $n\rho_{\mathrm{D}p}/\rho_{\mathrm{D}(p+4)}$, where $n \in \mathbb{Z}$ is the instanton number (second Chern class) of the gauge-field background. By inserting in (136), we learn that the $\mathrm{D}(p+4)$ brane has been endowed with n units of $\mathrm{D}p$ -brane charge. This suggests that the Higgs branch of the $\mathrm{D}p/\mathrm{D}(p+4)$ system is the moduli space of n -instanton solutions of theYang-Mills theory with gauge group $U(m)$. Note that in the absence of a first Chern class, i.e., if $\operatorname{tr}(F)=0$, the $\mathrm{D}p$ branes are only marginally bound to the $\mathrm{D}(p+4)$ branes.

> $\frac{1}{2}\operatorname{tr}(F\wedge F)$ 在 $\mathrm{D}(p+4)$ 膜的四个额外方向上的积分是一个拓扑不变量，等于 $n\rho_{\mathrm{D}p}/\rho_{\mathrm{D}(p+4)}$ ，其中 $n\in\mathbb{Z}$ 是规范场背景的瞬子数 (第二陈类)。将其代入 (136) 后我们得知，$\mathrm{D}(p+4)$ 膜带有 n 单位的 $\mathrm{D}p$ 膜电荷。这说明 $\mathrm{D}p/\mathrm{D}(p+4)$ 系统的希格斯分支就是规范群为 $U(m)$ 的杨-米尔斯理论中 n 瞬子解的模空间。注意，当不存在第一陈类时，也就是若 $\operatorname{tr}(F)=0$ ，$\mathrm{D}p$ 膜仅边缘束缚于 $\mathrm{D}(p+4)$ 膜。

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[28] If the four extra coordinates are non-compact, the $\mathrm{D}(p+4)$ worldvolume fields decouple from the $\mathrm{D}p$ -brane action, and $U(m)$ becomes a global symmetry.

> [28] 如果四个额外坐标是非紧致的，那么 $\mathrm{D}(p+4)$ 世界体积场会从 $\mathrm{D}p$ 膜作用量退耦，且 $U(m)$ 成为整体对称性。

The above statements can be made more precise. [29] The D-term equations (also called moment-map conditions) are the starting point of the celebrated Atiyah-Drinfeld-Hitchin-Manin (ADHM) construction of multi-instanton solutions [84]. And the metric on this moduli space is precisely the one induced by the flat $\mathbb{R}^{4n(m+n)}$ metric on the $V=0$ hypersurface quotiented by the action of $U(n)$.

> 上述结论可以更精确地表述。[29] D 项方程 (也称为矩映射条件) 正是著名的阿蒂亚-德林费尔德-希钦-曼宁 (ADHM) 多瞬子解构造的出发点 [84]。该模空间上的度量，正是由 $V=0$ 超曲面上的平直 $\mathbb{R}^{4n(m+n)}$ 度量商去 $U(n)$ 的作用后得到的。

The metric of the Coulomb branch is also curved, though this is not a classical effect but rather the result of quantum corrections. The annulus calculation of section ”Scattering D-Particles” for a D-particle scattering off a D4 brane gives indeed a $O\left(u^2\right)$ force which also receives non-perturbative corrections. For pure $N=2$ gauge theory, the Coulomb-branch metric is the celebrated Seiberg-Witten solution [85]. More Generally, its calculation is hard. The one certainty comes from supersymmetry which restricts the Coulomb-branch moduli space to be a special Kähler manifold and the Higgs-branch moduli space to be hyper-Kähler.

> 库仑分支的度量也是弯曲的，不过这不是经典效应，而是量子修正的结果。在“D 粒子散射”一节中，D 粒子被 D4 膜散射的圆环计算确实给出了一个 $O\left(u^2\right)$ 力，该力也会接收非微扰修正。对于纯 $N=2$ 规范理论，库仑分支度量就是著名的塞伯格-威滕解 [85]。一般来说它的计算非常困难。唯一确定的结论来自超对称: 超对称将库仑分支模空间限制为特殊凯勒流形，而希格斯分支模空间为超凯勒流形。

Special Effects

> ## 特殊效应

In an effort to learn more about quantum gravity, we considered up to now gauge theories that describe the dynamics of simple D-brane systems. Inverting the logic, one can try to engineer brane systems whose worldvolume theory is a quantum field theory of interest. This can offer several advantages: (i) geometrize moduli spaces and parameters; (ii) bring to light surprising connections between previously unrelated theories (one such example is 3*d* mirror symmetry [86,87]); and (iii) provide seeds of holographic dualities.

为了更深入研究量子引力，我们迄今为止讨论的都是描述简单 D 膜系统动力学的规范理论。反过来，我们也可以尝试构造膜系统，使其世界体积理论是我们感兴趣的量子场论。这有诸多优点:(i) 将模空间和参数几何化；(ii) 揭示此前不相关理论之间出乎意料的联系 (一个例子是 3*d* 镜对称 [86,87])；(iii) 为全息对偶提供基础。

Brane engineering is presented elsewhere in this volume (see also [88] for an early review). I close the present chapter with some special effects that play a role in these lego constructions. The first one is straightforward:

膜构造已在本卷其他章节介绍 (早期综述可参见 [88])。我在本章末尾介绍这类积木构造中发挥作用的一些特殊效应。第一个很直接:

Branes ending on branes As explained earlier, D-branes can dissolve as world-volume fluxes in higher-dimensional D-branes. But they can also end on other branes on whose worldvolumes they appear as gauge charges. This is obvious if we apply various U-dualities to a fundamental string suspended between two type-IIB D-branes. Consider, for example, the F-string stretching between two parallel D3 branes. Denote this configuration by [D3 → F1 → D3] and apply the following chain of dualities:

膜终止于膜如前文所述，D 膜可以以世界体积通量的形式溶解在更高维 D 膜中。但 D 膜也可以终止于其他膜，在这些膜的世界体积上表现为规范荷。如果对悬挂在两个 IIB 型 D 膜之间的基本弦应用各种 U 对偶，这一点就很明显了。例如，考虑伸展在两个平行 D3 膜之间的 F 弦，我们将这个构型记为 [D3 → F1 → D3]，对其应用如下对偶链:

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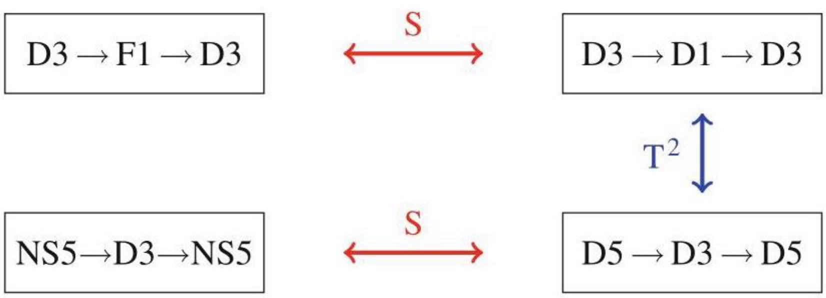
[29] As first noted in Refs. [81,82], for a nice review, see [83].

[29] 这一点最早由文献 [81,82] 指出，详细综述可参见 [83]。



From the S-duality on the top, we learn that D-strings can end on D3 branes. Such D-strings are the S-duals of charged gauge bosons, so they must correspond to the ’t Hooft-Polyakov monopoles of the world-volume $N = 4$ SYM. By T-dualizing two of the common transverse dimensions, we also learn that a D3 brane can end on D5 branes with which it shares two spatial dimensions. Finally, another S-duality shows that the D3 branes may also end on NS5 branes.

从最上方的 S 对偶我们可以得知，D 弦可以终止在 D3 膜上。这类 D 弦是带电规范玻色子的 S 对偶，因此它们必然对应世界体积 $N = 4$ SYM 的’t Hooft-Polyakov 磁单极子。对共同横向维度中的两个维度做 T 对偶，我们还可以得知，D3 膜可以终止在与它共享两个空间维度的 D5 膜上。最后，再一次 S 对偶表明，D3 膜也可以终止在 NS5 膜上。

Continuing like this, one can discover all other possibilities. Note that while U-dualities are a quick way to discover these rules, the fact that a brane X may end on a brane Y could be deduced ab initio from the possible gauge charges of the worldvolume theory of Y. As one example of creative engineering one can start from the lower-left end of the above duality chain, perform one more T duality to convert the D3 branes to D4 branes and the NS5 branes of type IIB to NS5 branes of type IIA, and add N_f D6 branes oriented so as not break extra supersymmetries. The resulting configuration can be lifted to M theory and realizes N $=$ 2 supersymmetric QCD with N_f flavors in four dimensions [89].

按照这种方式，我们可以推导出所有其他可能。需要注意的是，虽然 U 对偶是发现这些规则的快捷方式，但膜 X 可以终止在膜 Y 上这一结论，完全可以从 Y 的世界体积理论允许的规范电荷从头推导。作为创造性构造的一个例子，我们可以从上述对偶链的左下角出发，再做一次 T 对偶，将 D3 膜转化为 D4 膜，将 IIB 型的 NS5 膜转化为 IIA 型的 NS5 膜，再加入 N_f 个 D6 膜，调整其取向避免破坏额外超对称。得到的构型可以提升到 M 理论，实现四维带 N_f 味的 N = 2 超对称量子色动力学 [89]。

Brane creation and the s-rule Which brane can end on which other brane is a local rule. Each junction leaves 1/4 of the supersymmetries unbroken, but together the two suspending branes will break all supersymmetries unless they are carefully oriented in space, as in section ”D-Branes and Spacetime Supersymmetry.” There is however a robust phenomenon that happens when two D-branes share only one transverse dimension or only a longitudinal one, but not both. This is the generic situation for two planar D4 branes in $\mathbb{R}^9$, similar to two straight lines in $\mathbb{R}^3$.

膜产生与 s 规则哪个膜可以终止在哪个膜是一个局域规则。每个连接处都保留 1/4 的超对称性未破缺，但除非两个悬挂膜在空间中精心取向 (如章节“D 膜与时空超对称性”中的情况)，否则它们共同作用会破缺所有超对称性。当两个 D 膜仅共享一个横向维度，或仅共享一个纵向维度，而非同时共享一种维度时，会出现一个很稳健的现象。这是 $\mathbb{R}^9$ 中两个平面 D4 膜的一般情况，类似 $\mathbb{R}^3$ 中两条直线的情况。

When one D4 brane moves past the other, there could appear a tachyon indicating that the D-branes want to split and reconnect. Such instabilities are absent if the rotation from one D-brane to the other is sufficiently ”large,” for instance, if the D4 branes span orthogonal subspaces of $\mathbb{R}^9$. In this case, the stretched open strings have eight coordinates with mixed (ND) boundary conditions and 1/4-maximal supersymmetry is preserved. A T-dual configuration is the D8/D0 brane system which I will use to describe the phenomena at hand.

当一个 D4 膜经过另一个 D4 膜时，会出现快子，表明 D 膜想要发生分裂和重连。如果从一个 D 膜到另一个 D 膜的转动角度足够“大”，例如 D4 膜张成 $\mathbb{R}^9$ 的正交子空间，这种不稳定性就不会存在。此时，拉伸的开弦有八个坐标满足混合 (ND) 边界条件，保留 1/4 最大超对称性。我们接下来会用 D8/D0 膜系统的 T 对偶构型来描述这个现象。

Let the D8 brane extend in the directions $x^{j=2,\cdots,9}$ and consider the spectrum of the stretched string. In the NS sector, the coordinates $x^{j=2,\cdots,9}$ have half-integer modes and their fermionic partners integer modes. The ground state subtraction is thus minus that in Eq. (52), all the states are massive, and we are safely inside the stability regime. The Ramond ground state, on the other hand, is always massless.

设 D8 膜沿 $x^{j=2,\cdots,9}$ 方向延伸，我们来分析拉伸弦的能谱。在 NS 扇区中，坐标 $x^{j=2,\cdots,9}$ 具有半整数模，其费米子伙伴具有整数模。因此基态扣除量等于式 (52) 中扣除量的负值，所有态都是有质量的，我们完全处于稳定区间内。而朗道基态始终是无质量的。

Since there are only two fermionic zero modes, $\psi_0^{\mu=0,1}$, it is a two-dimensional Weyl fermion with space momentum replaced by the separation Δx^1 between the D-particle and the D8 brane. [30]

由于仅存在两个费米零模 $\psi_0^{\mu=0,1}$ ，这是一个二维外尔费米子，其空间动量被 D 粒子与 D8 膜之间的间距 Δx^1 替代。[30]

In the effective D8/D0 worldvolume theory, this open-string sector contributes therefore just a qubit: $|0\rangle$ if there is no such string or $|1\rangle$ if there is a string in its unique (supersymmetric) ground state. Any extra (D8-D0) strings must occupy excited states with masses $O\left(\sqrt{T_F}\right)$ and break supersymmetry.

在有效 D8/D0 世界体理论中，该开弦扇区仅贡献一个量子比特: 若弦处于其唯一的 (超对称) 基态则为 $|0\rangle$ if there is no such string or $|1\rangle$ 。任何额外的 (D8-D0) 弦都必须占据质量为 $O\left(\sqrt{T_F}\right)$ 的激发态，并破坏超对称。

Let us now see what happens when the D-branes cross. The Dirac-Weyl equation for the open-string fermion is $E = \pm T_F \Delta x^1$. Suppose that for $\Delta x^1 < 0$ the negative-energy solution of the Dirac sea is filled, so there is no string. As the D-particle crosses the D8 brane, the hole becomes positive energy out of the vacuum, see Fig. 10. This is the familiar story of the $2d$ chiral anomaly. At first sight, it looks as if $\Delta x^1 > 0$ is not a flat direction of the D-particle potential anymore since the energy of the created string grows linearly with separation. This conclusion is however wrong because the D8 brane generates a piecewise-linear background for the dilaton, $\Phi = 2\kappa_{10}^2 T_{\mathrm{D8}} \Theta\left(\Delta x^1\right) \Delta x^1$ with Θ the Heaviside step function. [31] This is computed using the effective actions of Section ”Effective Actions”. The mass of the D-particle, proportional in the string frame to $e^{-\Phi}$, is therefore reduced at leading order by

现在我们来看 D 膜交叉时会发生什么。开弦费米子的狄拉克-外尔方程为 $E = \pm T_F \Delta x^1$ 。假设对于 $\Delta x^1 < 0$ ，狄拉克海的负能解已被填满，因此不存在弦。当 D 粒子穿过 D8 膜时，空穴成为真空外的正能态，见图 10。这就是我们熟知的 $2d$ 手征反常。乍看之下，$\Delta x^1 > 0$ 似乎不再是 D 粒子势的平坦方向，因为生成弦的能量随间距线性增长。但该结论是错误的，因为 D8 膜为 dilaton 生成了分段线性背景，$\Phi = 2\kappa_{10}^2 T_{\mathrm{D8}} \Theta\left(\Delta x^1\right) \Delta x^1$ ，其中 Θ 是海维赛德阶跃函数。[31] 这一点可利用“有效作用量”章节的有效作用量计算得到。D 粒子的质量在弦框架下与 $e^{-\Phi}$ 成正比，因此领头阶会被降低为

$$T_{\mathrm{D0}}\Phi = T_F \Delta x^1, \tag{137}$$

where we have used the tension formula (94). This compensates precisely the energy of the stretched string, so Δx^1 is a modulus for both signs, as should be expected from the unbroken supersymmetry.

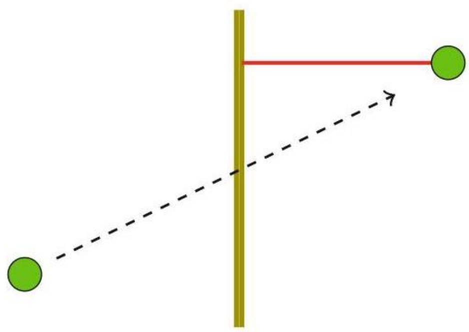
此处我们使用了张力公式 (94)。这恰好补偿了拉伸弦的能量，因此无论符号如何，Δx^1 都是模，这与未破缺超对称的预期一致。

These simple facts about the quantum mechanics of the D8/D0 system become highly nontrivial in other, non-perturbative duality frames. Consider, for example, the duality chain T(56) ST(789) where the T-dualized dimensions are shown in parentheses. It converts the D8/D0/F1 system to a D3 brane suspended between a D5 and a NS5 brane which have only one transverse dimension in common. If S-duality is valid, we conclude (i) that more than one D3 branes suspended between the two five-branes break all supersymmetries and (ii) that whenever a NS5 brane crosses a D5 brane, a D3 brane is created or destroyed depending on orientation.

D8/D0 系统量子力学的这些简单性质，在其他非微扰对偶框架中会变得高度非平凡。例如，考虑对偶链 T(56) ST(789)，其中括号内标注的是做 T 对偶的维度。它将 D8/D0/F1 系统转换为悬挂在 D5 膜与 NS5 膜之间的 D3 膜，且 D5 膜与 NS5 膜仅共享一个横向维度。若 S 对偶成立，我们可以得到结论:(i) 悬挂在两个五膜之间的多个 D3 膜会破坏所有超对称；(ii) 每当 NS5 膜穿过 D5 膜时，都会根据方向生成或湮灭一个 D3 膜。

Fig. 10 A D-particle crossing a D8 brane and creating a string (in red)

图 10 D 粒子穿过 D8 膜并生成一根弦 (红色所示)



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[30] A standard $2d$ fermion lives on the T-dual D9/D1 intersection. Note that the absence of a bosonic partner for the fermion is due to the fact that the unbroken supersymmetries act in the sector of opposite chirality, similarly to what happens on the worldsheet of the heterotic string.

[30] 标准 $2d$ 费米子存在于 T 对偶后的 D9/D1 交点上。请注意，费米子没有玻色伙伴是因为未破缺超对称作用在手征性相反的扇区，这与杂化弦世界面上的情况类似。

[31] We assume $\Phi = 0$ on the left where the observer of this gedanken experiment measures energy.

[31] 我们假设左侧满足 $\Phi = 0$，该思想实验的观测者在左侧测量能量。

These rules were first conjectured by Hanany and Witten [87] using plausibility arguments about gauge theories on D3 branes. Their connection with the Pauli exclusion principle and the $2d$ anomaly was shown in [90, 91] . Another interesting duality frame is the D6/D2/F1 system which lifts in M theory to a membrane in the background of the KK monopole; see section "Type IIA and M Theory." The Fermi-Dirac statistics of the open F-string translates in M theory to the fact that the classical membrane equations admit no holomorphic multi-membrane solutions [92]. This is a rather surprising check of the type-IIA/M-theory duality conjecture.

这些规则最初由 Hanany 和 Witten[87] 通过对 D3 膜上规范理论的合理性论证提出猜想。它们与泡利不相容原理和 $2d$ 反常的关联已在 [90, 91] 中证明。另一个有趣的对偶框架是 D6/D2/F1 系统，该系统在 M 理论中提升为 KK 单极背景下的膜；参见章节“IIA 型弦与 M 理论”。开 F 弦的费米-狄拉克统计在 M 理论中对应经典膜方程不存在全纯多膜解这一结论 [92]。这是对 IIA 弦/M 理论对偶猜想一个相当出人意料的验证。

Myers effect We have till now considered vanishing supergravity backgrounds. The study of D-branes in nontrivial geometries such as Calabi-Yau manifolds, [32] or in the context of AdS/CFT, is a rich subject in its own right. As an appetizer, I will conclude this chapter with the description of a general phenomenon with multiple manifestations and names: Myers or dielectric effect, giant gravitons, supertubes [42, 94-97] (see also [98, 99] for reviews). In a nutshell, this is the puffing up of D-branes into higher-dimensional ones that wrap topologically trivial cycles but are prevented from collapsing by background fields.

迈尔斯效应迄今为止我们一直讨论的是消失的超引力背景。对非平凡几何 (例如卡拉比-丘流形、[32]) 或 AdS/CFT 背景下 D 膜的研究本身就是一个内容丰富的课题。作为引子，我将在本章末尾介绍一个存在多种表现形式和名称的普遍现象: 迈尔斯效应又名介电效应、巨引力子、超管 [42, 94-97](综述另见 [98, 99])。简而言之，该现象是指 D 膜膨胀为更高维 D 膜，这类高维 D 膜缠绕拓扑平凡闭链，但背景场阻止其坍缩。

To illustrate the phenomenon, consider a spherical D2 brane in the background of a nonvanishing but weak electric 4-form flux. In locally flat polar coordinates, $F_4 \simeq fr^2 dr \wedge d(\cos\vartheta) \wedge d\varphi \wedge dt$ with f constant. We also switch on a worldvolume gauge field that endows the D2 brane with n units of D-particle charge. Choosing a convenient gauge for C_3 and in static coordinates for the D2 brane, we have

为说明这一现象，我们考虑非零弱电场 4 形式通量背景下的球形 D2 膜。在局部平坦极坐标中，$F_4 \simeq fr^2 dr \wedge d(\cos\vartheta) \wedge d\varphi \wedge dt$ 且 f 为常数。我们还开启了一个世界体积规范场，使 D2 膜带有 n 单位的 D 粒子荷。为 C_3 选取方便的规范，并为 D2 膜选取静态坐标后，我们得到:

$$C_3 \simeq \frac{fr^3}{3} \sin\vartheta d\vartheta \wedge d\varphi \wedge dt \quad \text{and} \quad F|_{D2} = \frac{n}{2T_F} \sin\vartheta d\vartheta \wedge d\varphi. \tag{138}$$

Inserting these fields in the D2-brane action Eq. (105) leads to the energy

将这些场代入 D2 膜作用量 (式 (105)) 可得能量:

$$E = 4\pi T_{\mathrm{D2}}\left[\sqrt{r^4 + \left(\frac{n}{2T_F}\right)^2} - \frac{1}{3}fr^3\right]. \tag{139}$$

We assume $f > 0$; otherwise, we change the D2 brane for an anti-D2 brane. The minimum of the energy is at the radius:

我们假设 $f > 0$； 否则我们将 D2 膜替换为反 D2 膜。能量的最小值出现在半径:

$$r_{\min} = \frac{nf}{4T_F} + O\left(f^5\right) \text{ with } E_{\min} = nT_{\mathrm{D0}}\left[1 - n^2 f^4 \frac{\left(\pi\alpha'\right)^2}{96} + O\left(f^8\right)\right],$$

(140)

where we used $2\pi nT_{\mathrm{D2}}/T_F = nT_{\mathrm{D0}}$. One sees that for $f = 0$ the D2 brane shrinks to a point and its energy is that of n D-particles. But the nonvanishing F_4 exerts an outward pressure which combines with the pressure of the magnetic field to puff up the D2 brane to a sphere. Note that both f and n are necessary to stabilize the brane at nonzero radius.

其中我们用到了 $2\pi nT_{\mathrm{D2}}/T_F = nT_{\mathrm{D0}}$ 。可以看出，当 $f = 0$ 时，D2 膜收缩为一个点，其能量等于 n 个 D 粒子的能量。但非零的 F_4 会产生向外的压强，该压强与磁场压强结合使 D2 膜膨胀为球形。注意，要使膜稳定在非零半径，f 和 n 二者缺一不可。

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[32] For a review, see e.g.,[93].

[32] 综述可参见例如文献 [93]。

It is instructive to consider the D0-brane perspective. In the absence of the RR background the low-energy Hamiltonian of the n D-particles is the reduction to 0+1 dimensions of $N = 4$ SYM with gauge group $U(n)$. We choose now Cartesian local coordinates in which $C_3 \simeq fx^1 \wedge dx^2 \wedge dx^3 \wedge dt$. This background adds a new term to the potential [95] which restores the invariance of the non-abelian extension of the D2-brane coupling $T_{\mathrm{D2}} \int (C_3 + F \wedge C_1)$ under the T-duality of X^2, X^3 that exchanges the D2 branes with D-particles, the RR 3-form with the 1-form, and the D2-brane gauge field with the D-particle position. The potential to lowest order reads

从 D0 膜的角度思考会很有启发。在没有 RR 背景的情况下，n 个 D 粒子的低能哈密顿量是规范群为 $U(n)$ 的 $N = 4$ 超杨-米尔斯理论约化到 0+1 维的结果。现在我们选取满足 $C_3 \simeq fx^1 \wedge dx^2 \wedge dx^3 \wedge dt$ 的笛卡尔局部坐标。该背景为势能添加了一个新项 [95]，这项恢复了 D2 膜耦合 $T_{\mathrm{D2}} \int (C_3 + F \wedge C_1)$ 的非阿贝尔扩展在 X^2, X^3 的 T 对偶下的不变性——该 T 对偶交换 D2 膜与 D 粒子、RR 3 形式与 1 形式，以及 D2 膜规范场与 D 粒子位置。最低阶的势能为:

$$V = T_{\mathrm{D0}}\,\mathrm{tr}\left(-\frac{1}{4}T_F^2\left[Y^i, Y^j\right]\left[Y^i, Y^j\right] + \frac{i}{6}T_F f \varepsilon_{ijk} Y^i\left[Y^j, Y^k\right]\right) \tag{141}$$

where repeated indices are implicitly summed and the coefficients are fixed by the appropriately normalized open-string coupling. [33]

其中重复指标默认为求和，系数由适当归一化的开弦耦合确定。[33]

Varying the above potential gives the equations

对上述势能变分可得方程:

$$\left[\left[Y^i, Y^j\right] - i\frac{f}{2T_F}\varepsilon_{ijk}Y^k, Y^j\right] = 0, \tag{142}$$

which can be solved by matrices that obey the $su(2)$ algebra $\left[Y^i, Y^j\right] = \ i(f/2T_F)\varepsilon_{ijk}Y^k$. The solution that minimizes the energy is the one that maximizes the $su(2)$ Casimir, so it corresponds to the irreducible n -dimensional representation of the Lie algebra. Inserting back in (141), we find

它可以通过满足 $su(2)$ 代数 $\left[Y^i, Y^j\right] = \ i(f/2T_F)\varepsilon_{ijk}Y^k$ 的矩阵求解。使能量最小的解对应最大化 $su(2)$ 卡米尔量的解，因此它对应李代数的不可约 n 维表示。代回式 (141)，我们得到

$$\sum_{i=1}^{3} Y^i Y^i = \frac{f^2\left(n^2-1\right)}{16T_F^2}\mathbf{1}_{n\times n},\ V_{\min} = -T_{D0}\frac{f^2}{24}\operatorname{tr}\left(\sum_{i=1}^{3} Y^i Y^i\right). \tag{143}$$

The D-particles have thus puffed up to the fuzzy (alias noncommutative) sphere with radius $\simeq fn/4T_F$ for n large. These results are in perfect agreement with Eq. (140). Note that computing subleading terms in the f -expansion would require the non-abelian generalization of the DBI action which is plagued by ordering ambiguities; see, e.g., [98].

因此 D 粒子膨胀为半径为 $\simeq fn/4T_F$ 的模糊 (即非对易) 球面，其中 n 很大。这些结果与式 (140) 完全一致。注意，计算 f 展开中的次领头项需要对 DBI 作用量做非阿贝尔推广，而该推广存在排序歧义；参见例如文献 [98]。

NS5 brane synthesis We conclude with an example that illustrates the effects of this subsection all at once [42, 100]. It has also the advantage of being exact to all orders in α' which means that it can be handled with the methods of boundary conformal field theory (BCFT).

NS5 膜合成我们最后用一个例子一次性说明本小节的所有效应 [42, 100]。它还有一个优势: 结果对 α' 的所有阶都精确，因此可以用边界共形场论 (BCFT) 方法处理。

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[33] We did not bother with this normalization up to now, but it is needed for the non-abelian terms in the D-brane actions. One quick way to determine it is by relating the instanton number on a D4 brane to the number of D-particles; see section ”Bound States and Moduli Spaces.”

[33] 到目前为止我们都没有考虑这个归一化，但它对于 D 膜作用量中的非阿贝尔项是必需的。确定它的一个简便方法是将 D4 膜上的瞬子数与 D 粒子数联系起来；参见章节“束缚态与模空间”。

The background is the near-horizon region of k type-IIB NS5 branes that we have briefly encountered in the duality section "Dualities." This is an exact worldsheet CFT made of three parts: (i) free string coordinates $X^{\mu=0,\cdots,5}$ along the worldvolume of the five-branes; (ii) a Wess-Zumino-Witten (WZW) model whose target space is the round 3-sphere that surrounds the branes; and (iii) a linear-dilaton CFT for the radial direction $\log|X^{\perp}|$ [53]. The metric and 2-form field of the WZW model read

该背景是我们在对偶章节“对偶性”中简要提及的 k 个 IIB 型 NS5 膜的近邻区域。这是一个精确的世界面 CFT，由三部分组成:(i) 沿五膜世界体积的自由弦坐标 $X^{\mu=0,\cdots,5}$ ；(ii) 目标空间为包围五膜的圆三维球面的 Wess-Zumino-Witten(WZW) 模型；(iii) 径向方向 $\log|X^{\perp}|$ 的线性 dilaton CFT[53]。WZW 模型的度规和二形式场为

$$ds^2 = k\alpha' \left(d\psi^2 + \sin^2\psi d\Omega^2\right) \quad \text{and} \quad B_2 = k\alpha' \left(\psi - \frac{1}{2}\sin 2\psi\right)\omega_2, \tag{144}$$

where $d\Omega^2$ and ω_2 are the metric and the area form of the 2-sphere of unit radius. The field strength $H_3 = dB_2$ is the volume form of the 3-sphere; the gauge potential B_2 has a Dirac singularity at the south pole, $\psi = \pi$, but this cannot be detected by fundamental strings as long as k is an integer.

其中 $d\Omega^2$ 和 ω_2 分别是单位半径二维球面的度规和面积形式。场强 $H_3 = dB_2$ 是三维球面的体积形式；规范势 B_2 在南极存在狄拉克奇点 $\psi = \pi$ ，但只要 k 为整数，基本弦就无法探测到该奇点。

Consider now n coincident D3 branes ending on the NS5 branes and preserving 1/4 supersymmetry. These must be oriented as described in the beginning of this subsection, i.e., they span the radial direction as well as two of the NS5-brane directions, say x^1, x^2 . They intersect the 3-sphere at a point (which we choose without loss of generality to be the north pole $\psi = 0$). Nothing prevents, however, the D3 branes from puffing up into a D5-brane if this can lower the energy. In addition to the spectator dimensions $x^1, x^2, |x^{\perp}|$, the D5 brane must now wrap a 2-sphere at some polar angle ψ of the 3-sphere, as depicted in Fig. 11 on the left. Furthermore, a worldvolume magnetic field must endow the D5 brane with n units of D3-brane charge. Since there are no RR backgrounds in the problem, the energy is given entirely by the DBI part of the D5-brane action. It is proportional to

现在考虑 n 个重合 D3 膜端点在 NS5 膜上，且保留 1/4 超对称。它们必须按本小节开头所述的方式取向: 即同时张成径向方向和两个 NS5 膜方向，记为 x^1, x^2 。它们与三维球面交于一点 (不失一般性，我们取该点为北极点 $\psi = 0$)。但如果膨胀为 D5 膜可以降低能量，那么没有什么能阻止 D3 膜膨胀成 D5 膜。除观测维度 $x^1, x^2, |x^{\perp}|$ 外，D5 膜现在必须缠绕三维球面中某个极角 ψ 处的二维球面，如图 11 左侧所示。此外，世界体积磁场必须给 D5 膜赋予 n 单位的 D3 膜电荷。由于该问题中不存在 RR 背景，能量完全由 D5 膜作用量的 DBI 部分给出，它正比于

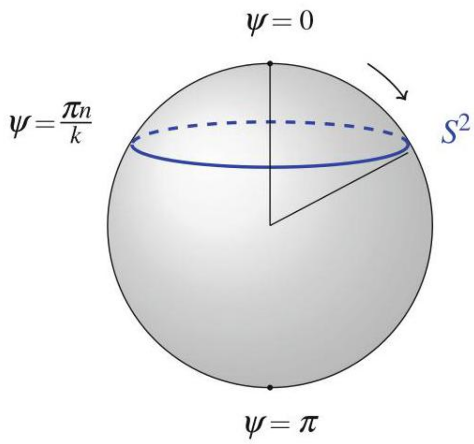
$$E \propto k\alpha' \left[\sin^4\psi + \left(\psi - \frac{1}{2}\sin 2\psi - \frac{\pi n}{k}\right)^2\right]^{1/2}. \tag{145}$$

We have used here the appropriately normalized worldvolume flux $F = -n\omega_2/2T_F$. The minimum of (145) is at $\psi_0 = \pi n/k$, so for n small the D3 branes puff up to a D5 brane of radius $\sim \psi_0\sqrt{k\alpha'}$. This is a variant of the Myers effect.

我们此处使用了经过适当归一化的世界体通量 $F = -n\omega_2/2T_F$ 。(145) 式的最小值位于 $\psi_0 = \pi n/k$ ，因此当 n 较小时，D3 膜会膨胀为半径为 $\sim \psi_0\sqrt{k\alpha'}$ 的 D5 膜。这是迈尔斯效应的一个变体。

Fig. 11nD3 branes puffing up to a D5 brane in the near-horizon of k NS5 branes

图. 11nD3 在 k 个 NS5 膜的近视界区域膨胀为 D5 膜的膜



But something else happens now as n keeps increasing. The D5 brane crosses the equator and then starts shrinking again untill, when $n = k$, it becomes a point at the south pole. To understand this phenomenon, note that in the asymptotically flat geometry the 3-sphere radius diverges and the D5 brane opens up to an infinite hyperplanar brane. This is the [kNS5 → nD3 → D5] configuration of Hanany and Witten, with the back-reaction of the NS5 brane on the supergravity fields taken into account. The fact that n takes values in $[0, k]$ is our s-rule. Furthermore, when the asymptotically planar D5 brane crosses to the other side of the NS5 branes, its trajectory wraps the 3-sphere from north to south creating k D3 branes. Finally, n transforms under large gauge transformations of B_2 (intuitively, these add extra NS5 branes at infinity). It is the Page charge defined in Eq. (106). [34]

但当 n 持续增大时，情况发生了变化:D5 膜穿过赤道后再次收缩，当 $n = k$ 时，它收缩为南极点上的一个点。要理解该现象，注意在渐近平直几何中，三维球半径发散，D5 膜会张开成为一张无限的超平面膜。这就是 Hanany 和 Witten 提出的 [kNS5 → nD3 → D5] 构型，其中已经考虑了 NS5 膜对超引力场的背景反应。n 取值于 $[0, k]$ 这一性质就是我们所说的 s-规则。此外，当渐近平直的 D5 膜运动到 NS5 膜另一侧时，它的轨迹从北极到南极包裹住三维球，生成了 k 个 D3 膜。最后，n 在 B_2 的大规范变换下发生变换 (直观来说，这些变换会在无穷远处增加额外的 NS5 膜)，这正是 (106) 式定义的佩奇电荷。[34]

The puffed-up D5 branes in the above example have an exact CFT description as Cardy states of the $\widehat{su}(2)_k$ WZW model [102]. From the perspective of the D3 branes, this is a reincarnation of a celebrated phenomenon in condensed-matter physics, the screening of a magnetic spin- $(n-1)/2$ impurity in the k -channel Kondo model; see [103, 104].

上述例子中膨胀得到的 D5 膜，作为 $\widehat{su}(2)_k$ WZW 模型的卡迪态，存在精确共形场论描述 [102]。从 D3 膜的视角来看，这是凝聚态物理中一个著名现象的重现: 即 k 通道近藤模型中磁自旋- $(n-1)/2$ 杂质的屏蔽；参见 [103, 104]。

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[34] A mathematically precise definition of the Page charge in this coontext can be given in terms of twisted K-theory [101].

[34] A 在本文背景下，佩奇电荷可以用扭曲 K 理论给出数学上精确的定义 [101]。

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